Nonlinear Modes of Vibration of Beams and Plates

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Abstract. It is intended in this lecture to analyse modes of vibration in the non-linear regime. After an introduction, where the concept of mode of vibration of non-linear structures is discussed, the main assumptions and a procedure to derive ordinary differential equations that represent structural elements are presented. For this purpose, we first focus on the case of beams vibrating in one plane; later on, the plate case is introduced. Non-linearity induced by large displacements is taken into account in the strain-displacements relations. We also address the inclusion of plastic strains on equations of motion. The Principle of Virtual Work is applied, with the Principle of d'Alembert, to obtain a system of ordinary differential equations that represents the structure under analysis. Writing the time functions as truncated Fourier series, sets of algebraic equations in the frequency domain are derived. A continuation method is employed to solve the ensuing algebraic equations of motion. The presentation proceeds to the analysis of periodic, conservative, free vibrations of beams and plates. The variation of frequency and shape of vibration with the vibration amplitude is investigated. Internal resonances, that is, couplings between modes that are induced by the non-linearity, are also investigated. These resonances lead to rather interesting vibrations.