Variance reduction schemes for Monte Carlo estimators in global illumination algorithms

László Szécsi

Budapest University of Technology

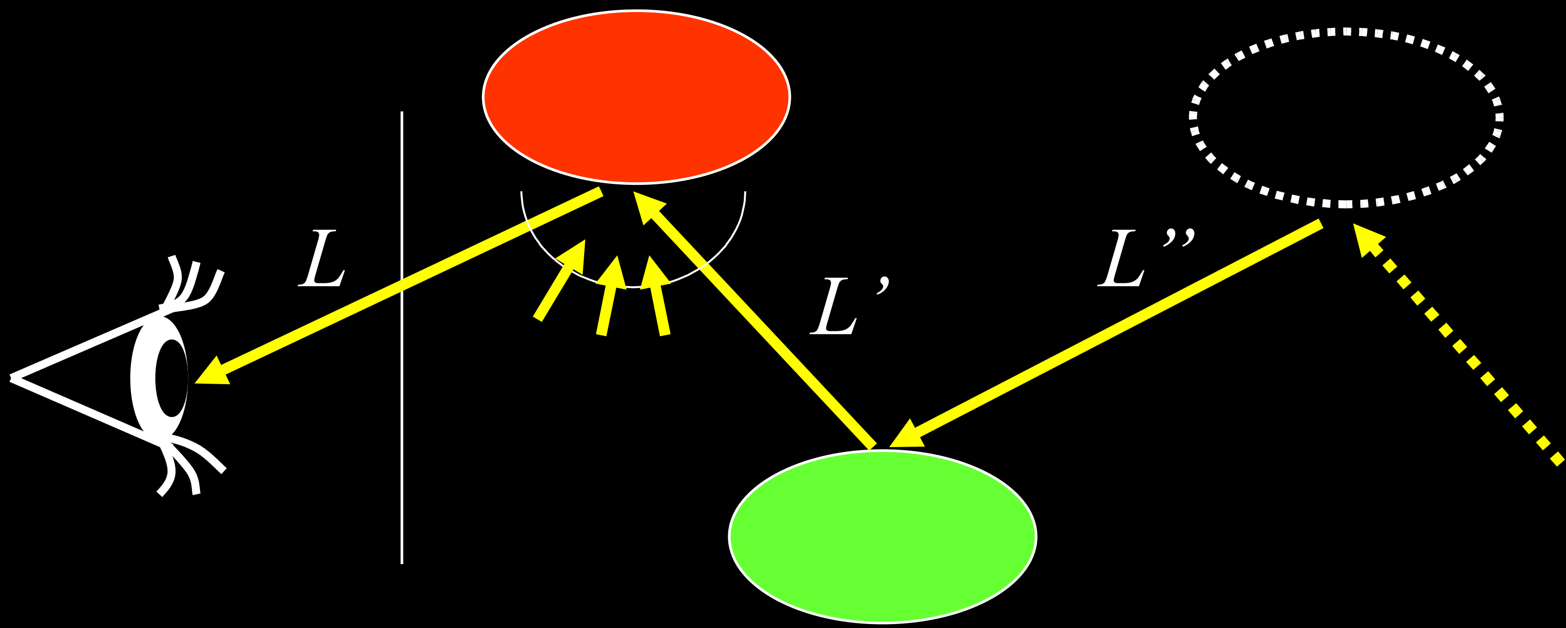
Scope of this talk

- the rendering problem
  - as an integral equation
- examples from our previous work
  - how Monte-Carlo variance reduction techniques translate to better global illumination rendering algorithms
- overview
  - instead of detailed analysis

The rendering problem

- Find the radiance toward the eye from surface element visible in pixels

$$L(\omega) = \int w(\omega', \omega)\, L'(\omega')\, \mathrm{d}\omega'$$

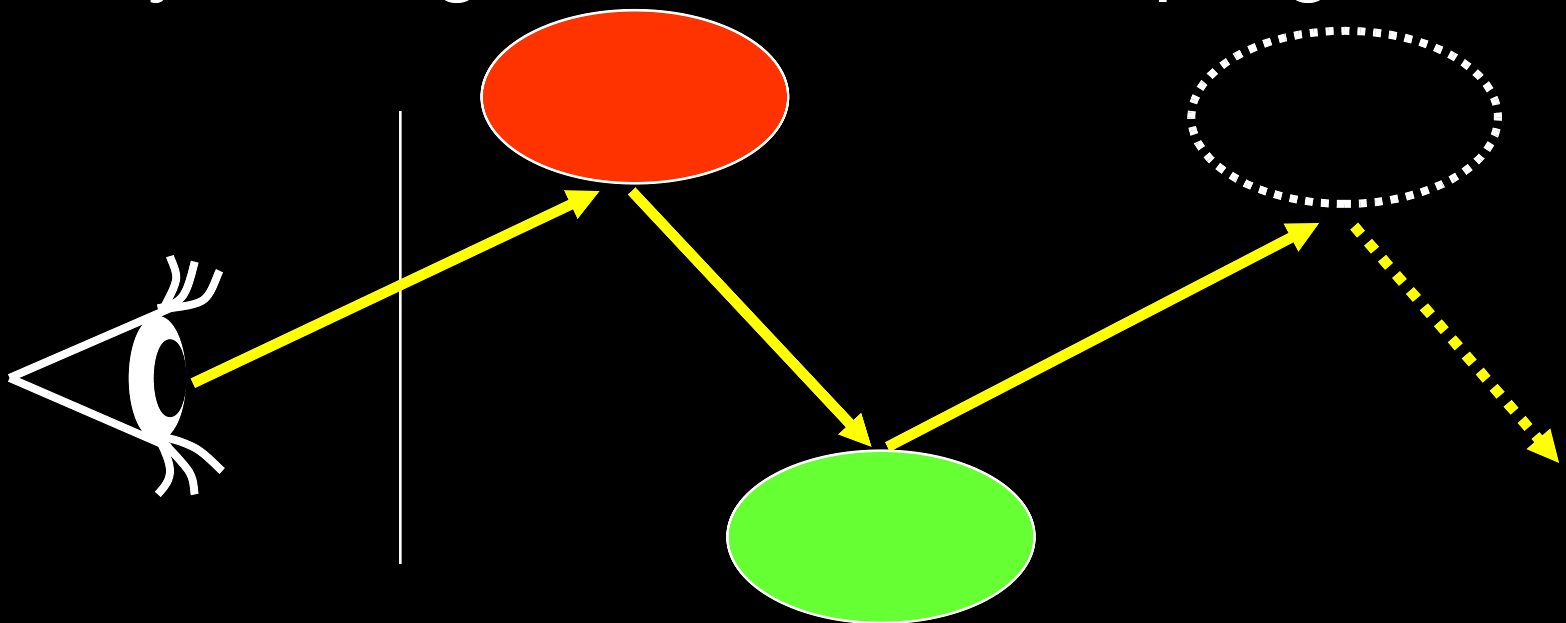


Random walk

- Monte-Carlo integration

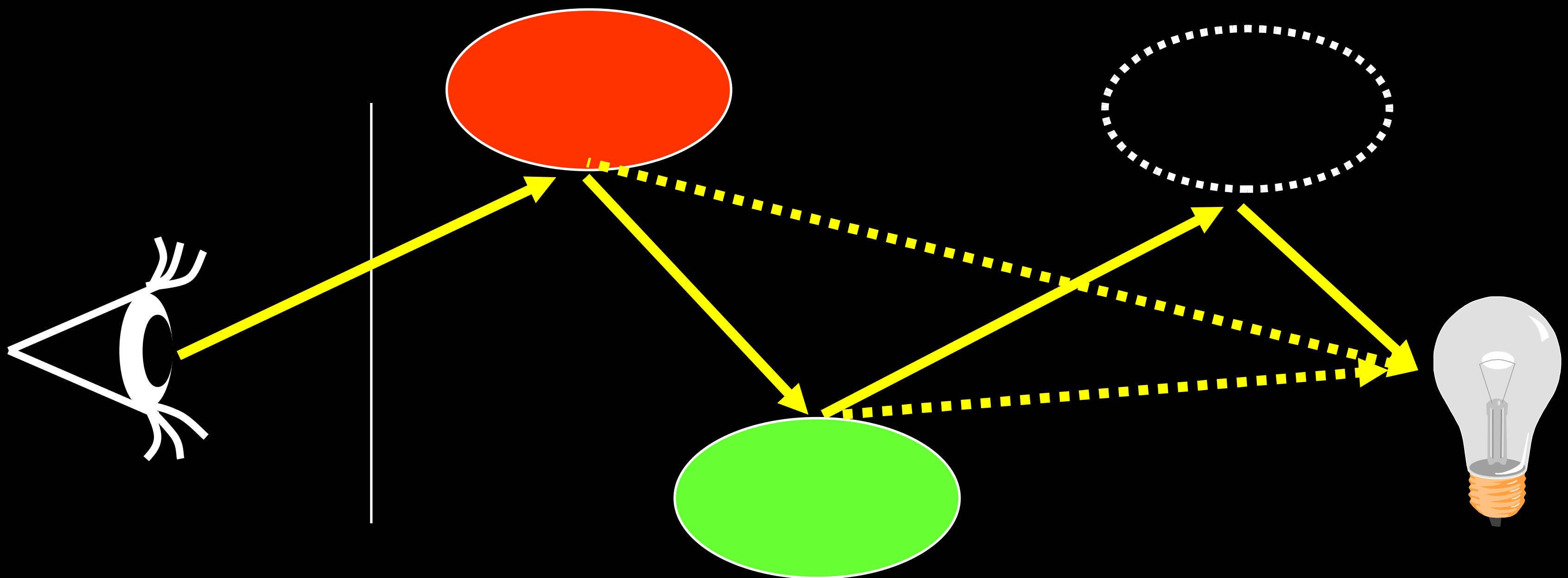
$$L = E\left[\frac{w(\omega')\,L'(\omega')}{p(\omega')}\right]$$

- Ray casting + directional sampling

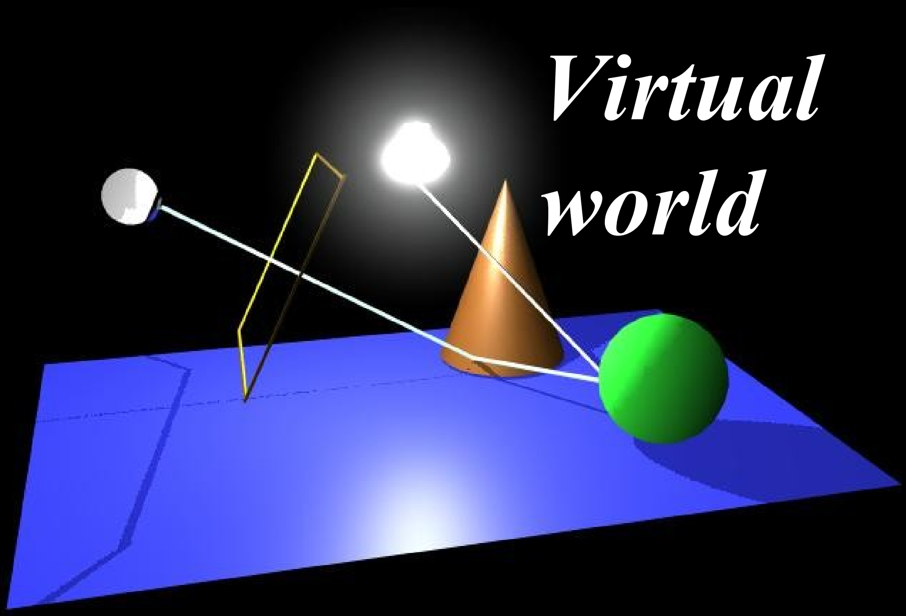


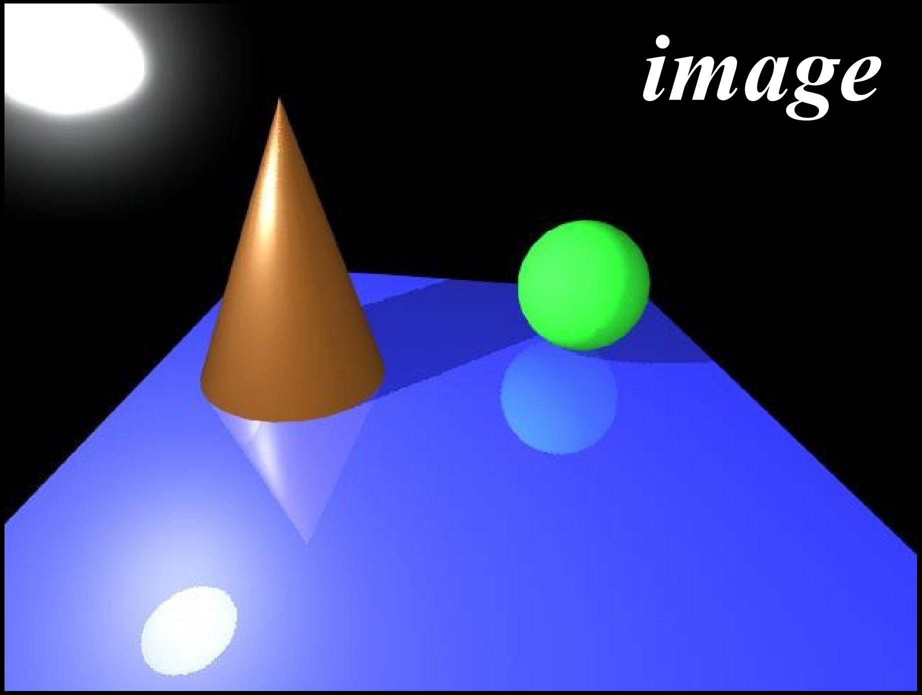
Termination

- approximate incoming radiance with direct illumination only (next event estimate)
- connect light path to light source



GI rendering = light path generation



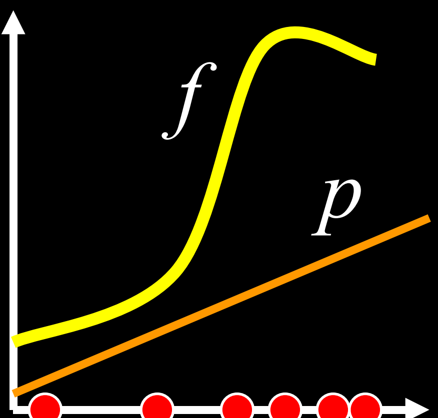


Efficiency issues

- Path space has high dimension
  - Low discrepacy sampling: (quasi) Monte Carlo
- Concentrate on large contribution paths
  - Importance sampling
- Computational cost of a single path
  - Path reuse

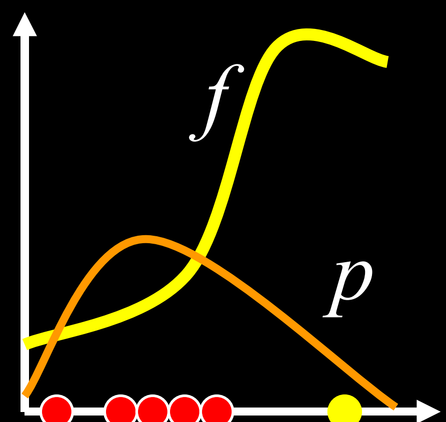
Importance sampling

Estimate: $\int f(x)\,\mathrm{d}x \approx 1/M \sum f(x_i)/p(x_i)$



good

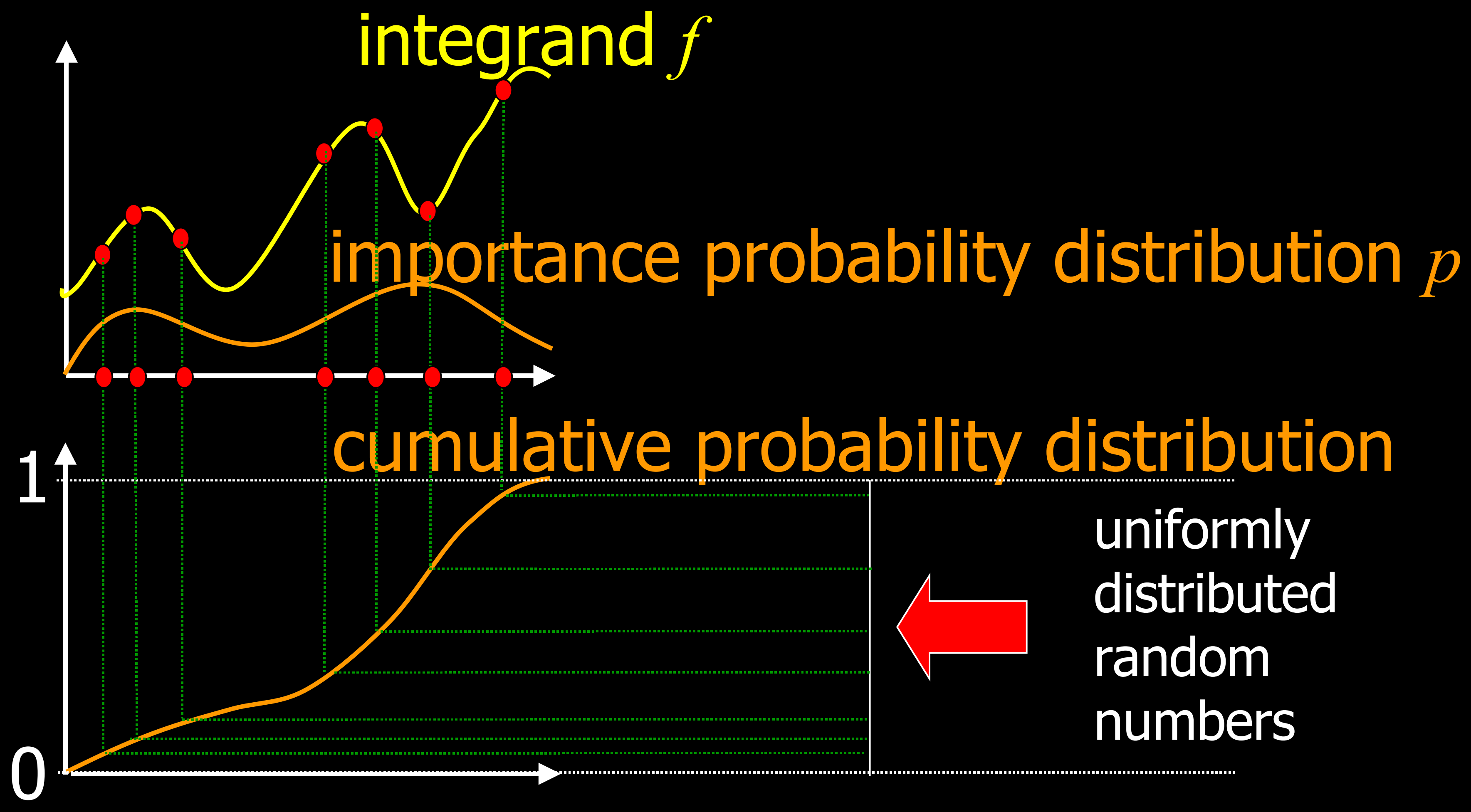
similar f/p samples



bad

a few, but large f/p samples

Sample generation



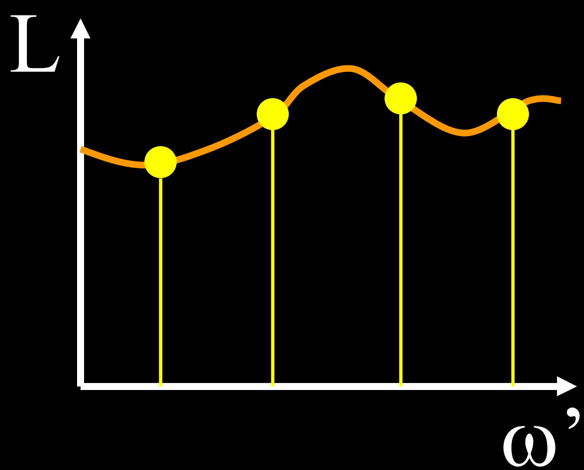
Example 1

Light path termination

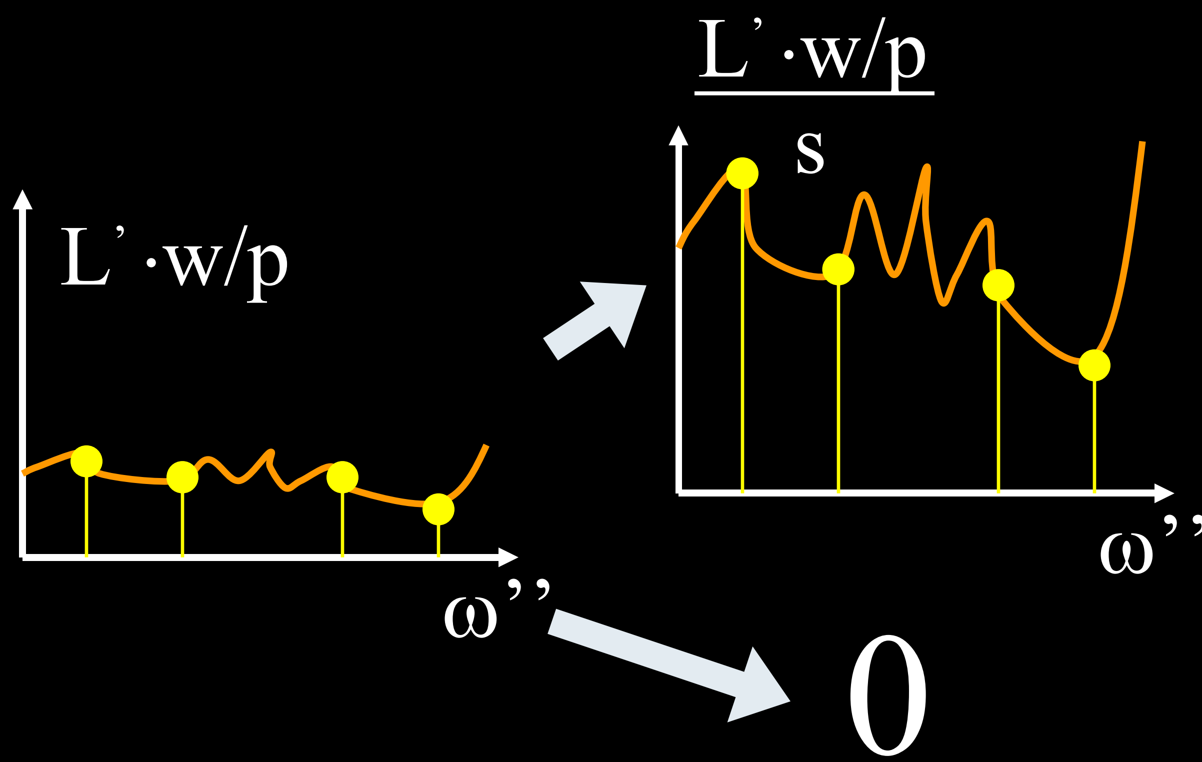
Russian Roulette

Roussian-roulette

contribution of light path length n

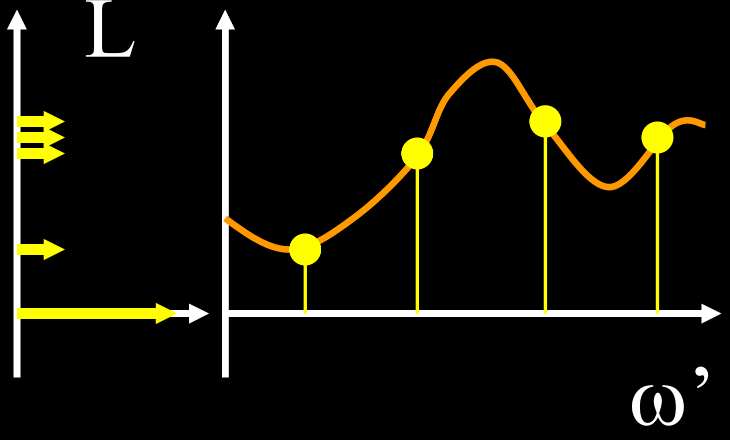


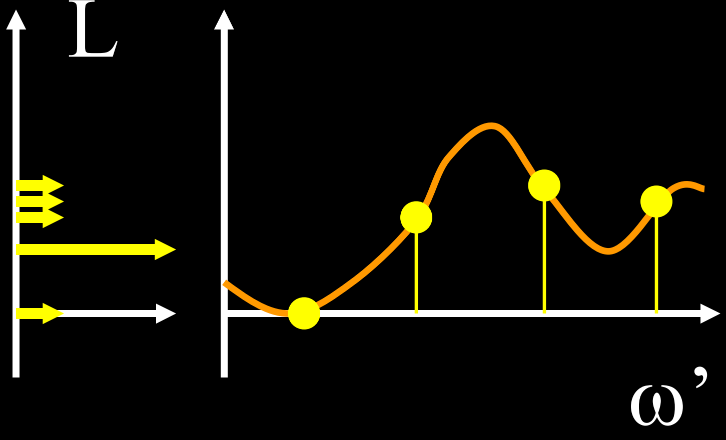
contribution of light path length n+1



Terminal estimate

- Estimator is zero if the walk is not continued
  - Variance increase
- Use some rough estimate instead

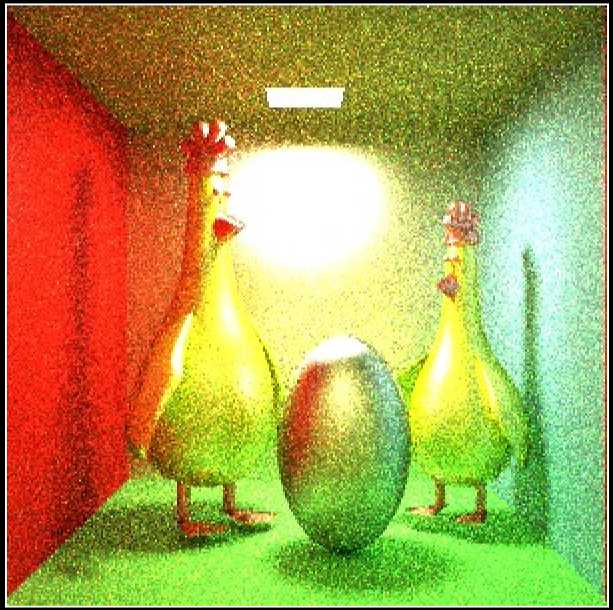




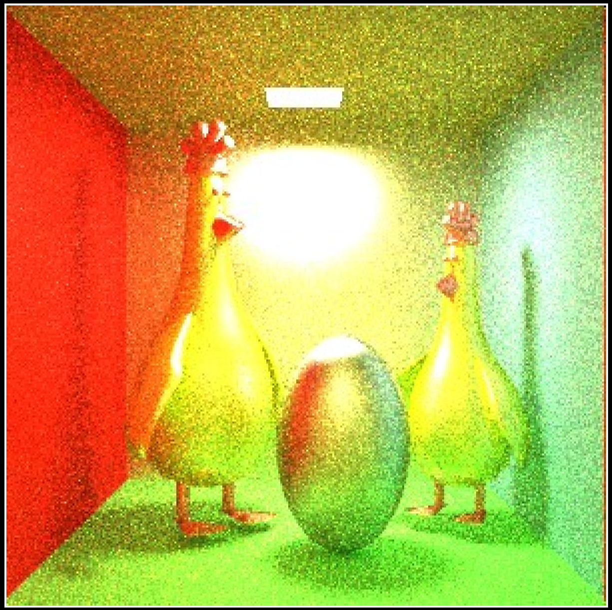
Terminal estimate

- Globally
  - Power is multiplied after every reflection by the average albedo
  - Total power in the scene is the sum of a geometric series
- Locally
  - Cheap approximate radiance computation method

Results



Classic RR

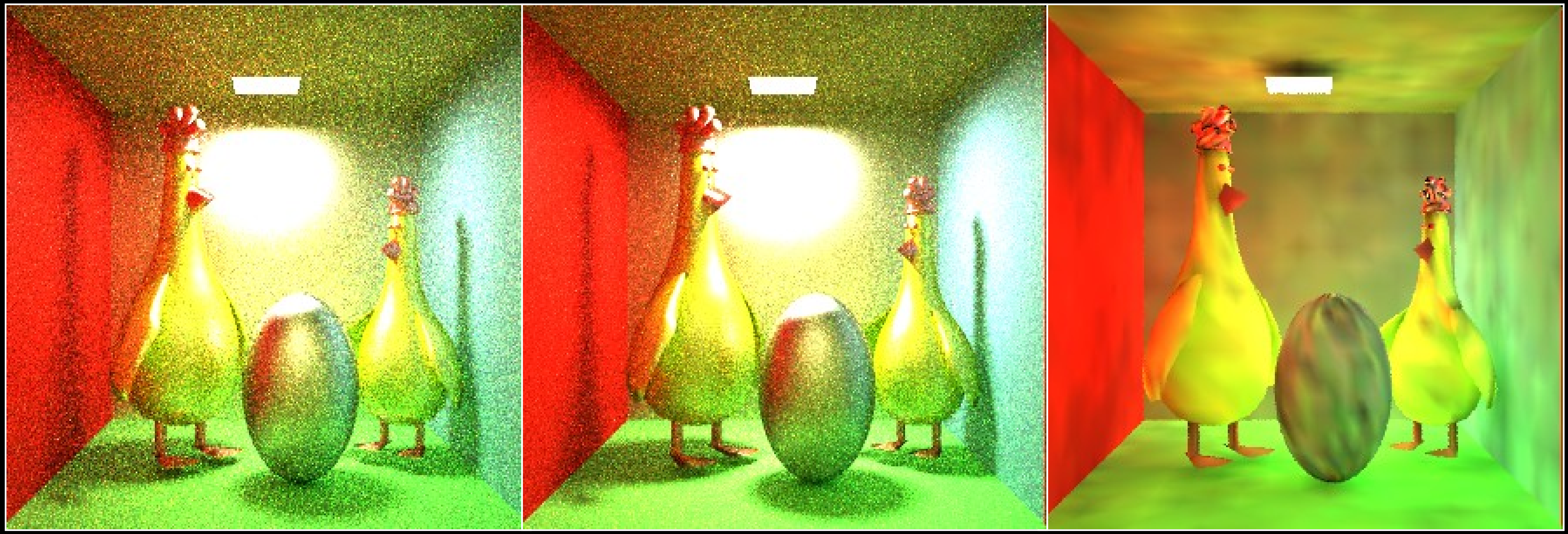


Improved RR

Up to 30% speedup

Local guess

- Finite element shooting walk algorithm
  - negligible time cost



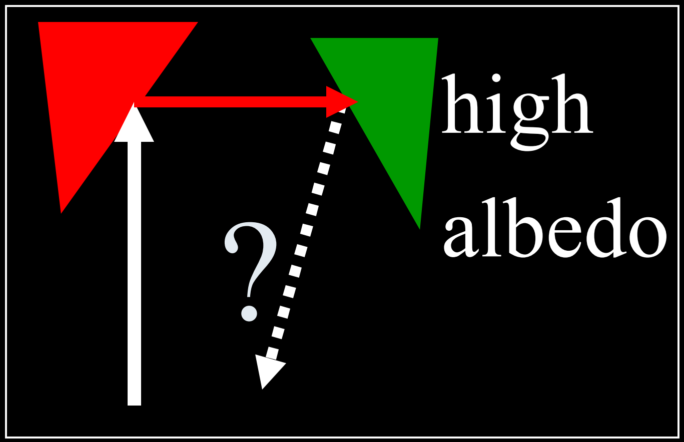
Classic RR Improved RR Terminal guess

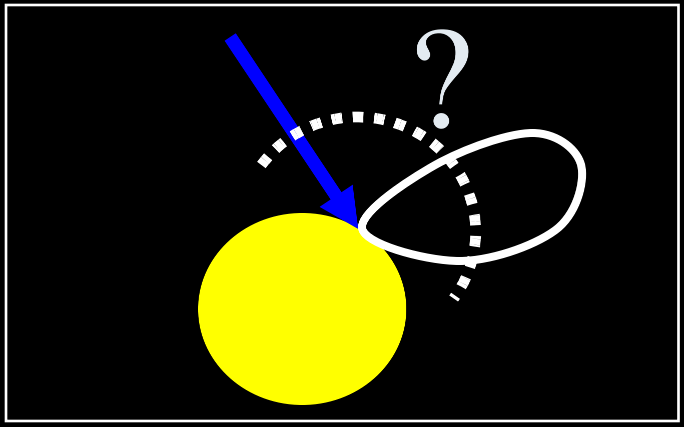
Example 2

Spectral optimisation
for path termination

Spectral optimisation

- Red light arrives on green wall
- Blue light arrives on yellow plastic with white specular
  - high diffuse albedo
  - no diffuse reflection

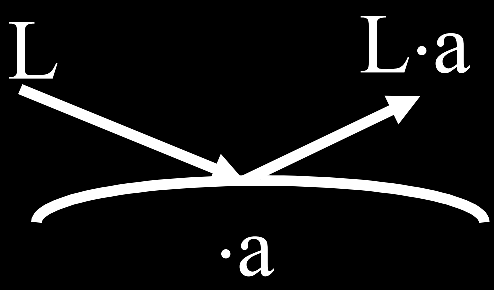




Spectral optimisation

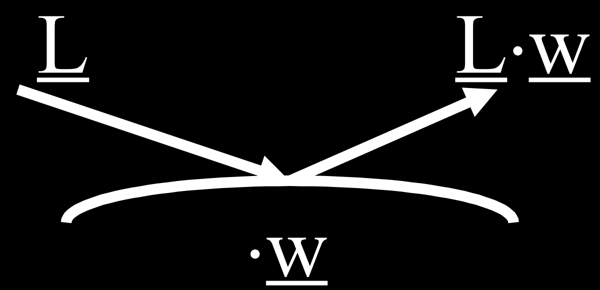
- Importance sampling
  - keep estimator value constant

Scalar value



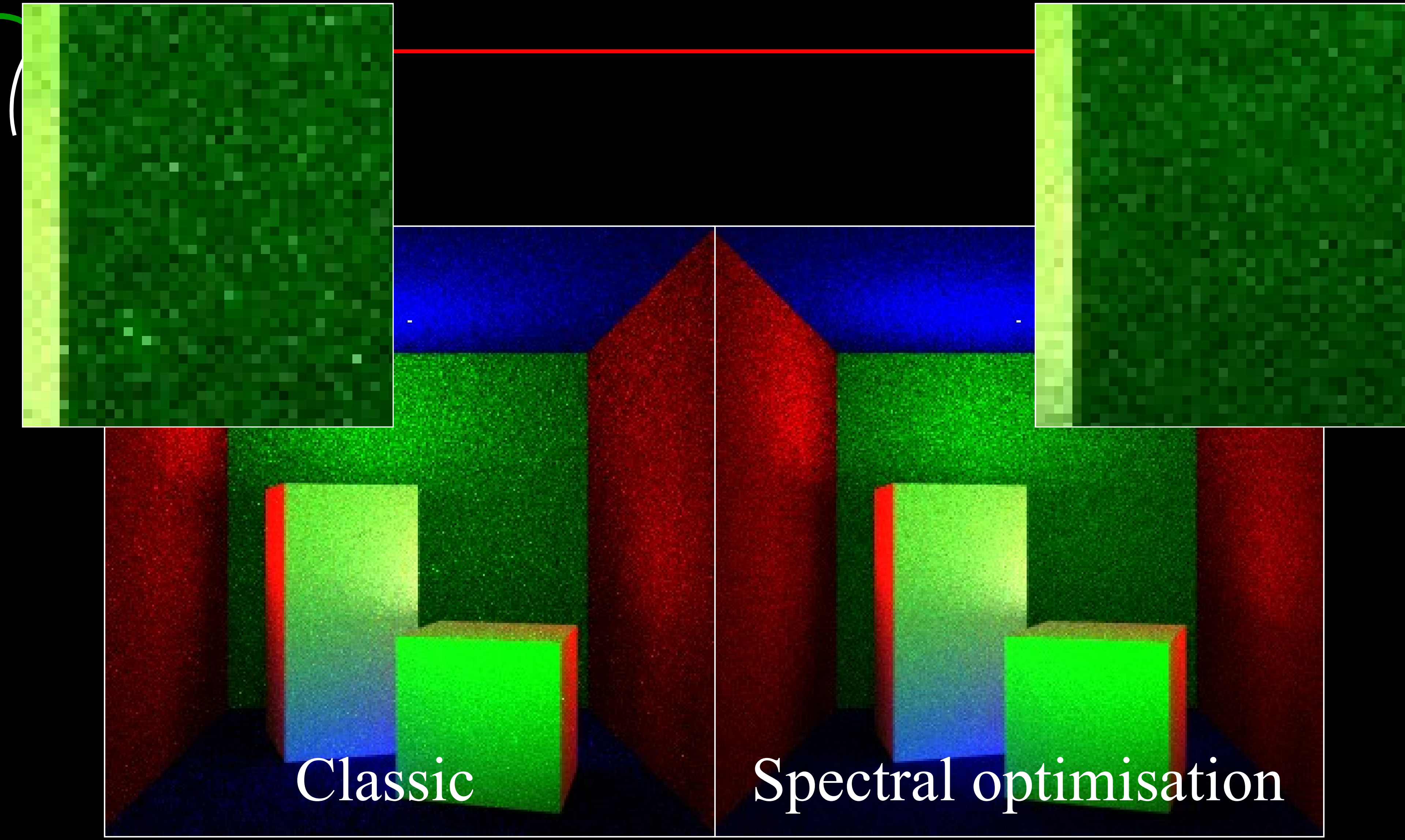
$s = a$

Vector value



$$s = \frac{\Sigma L_\lambda \cdot w_\lambda}{\Sigma L_\lambda}$$

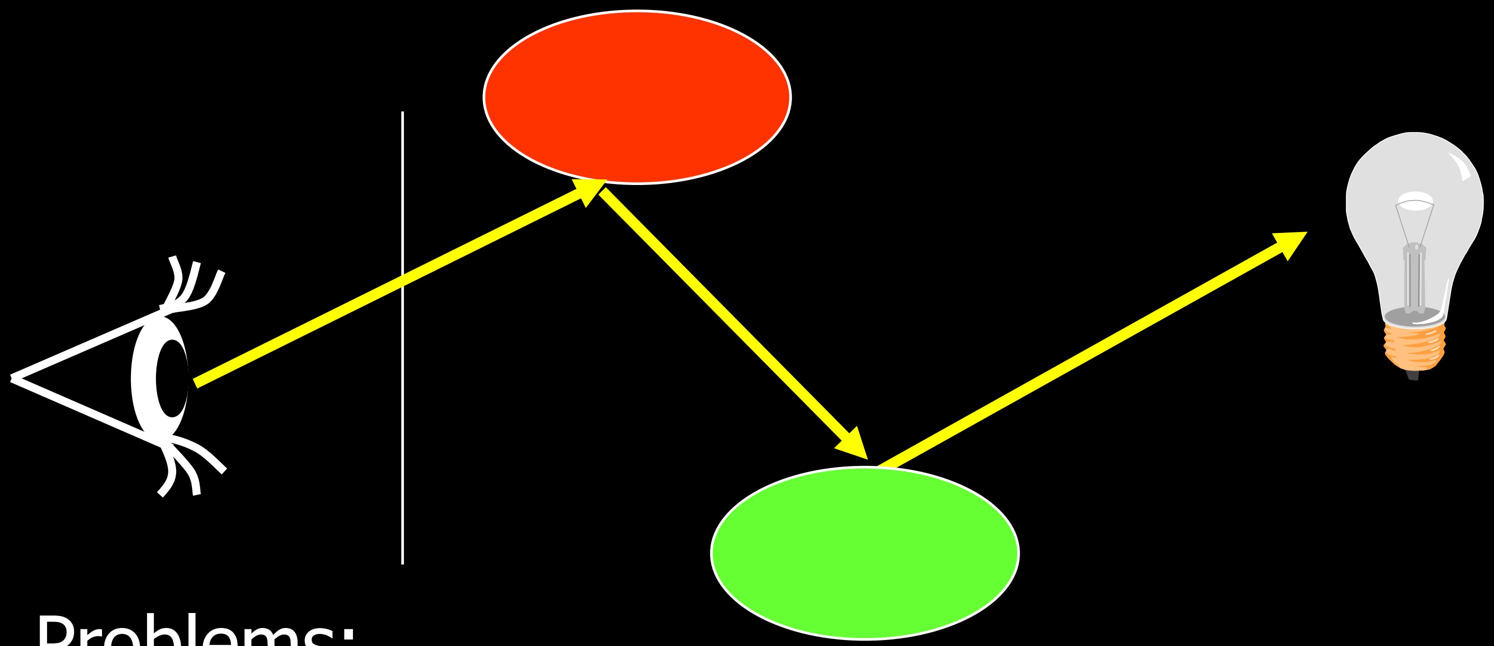
Spectral optimisation



Example 3

”Go with the Winners” in Path Tracing

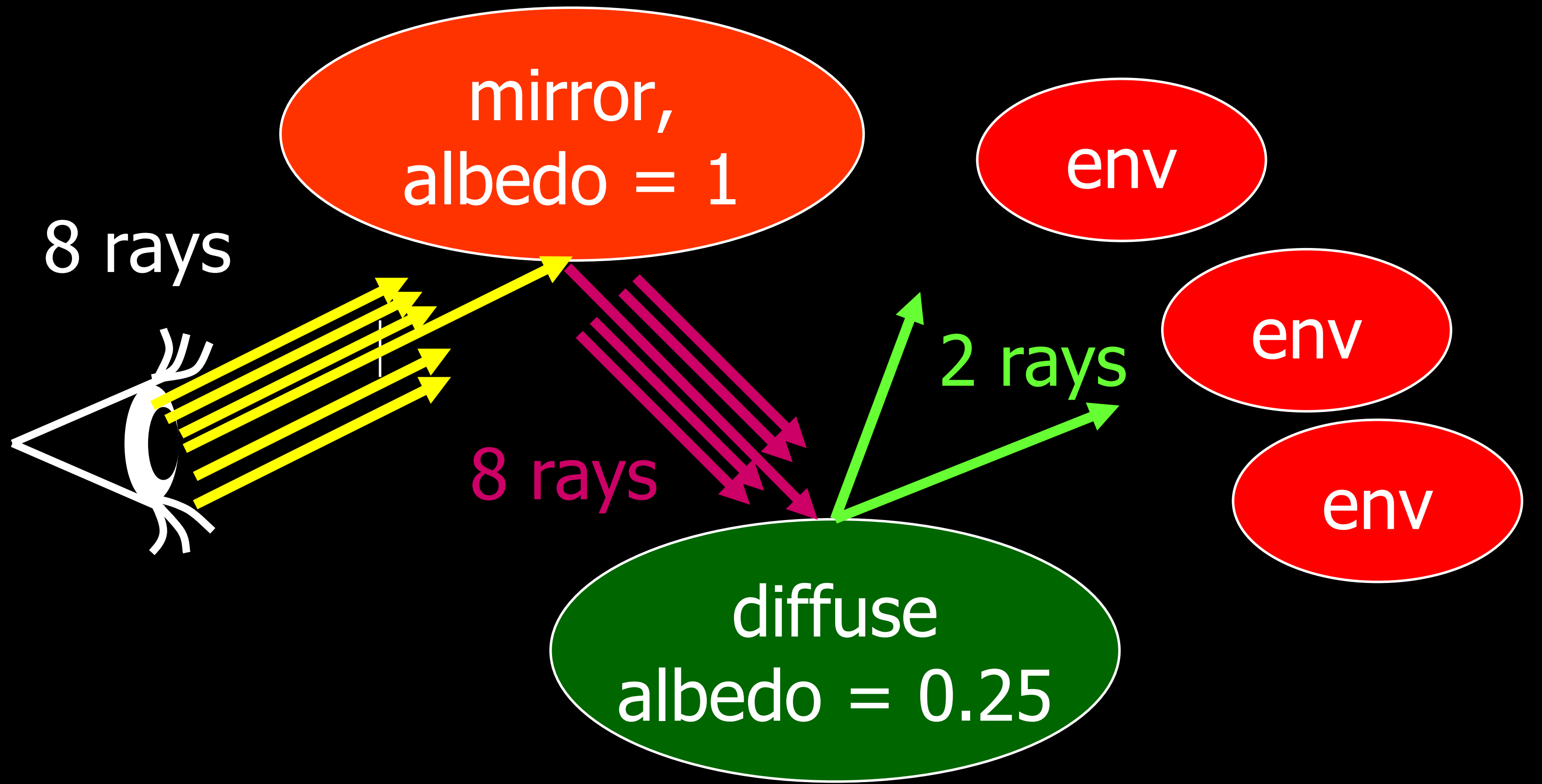
Path tracing with Russian-roulette



Problems:

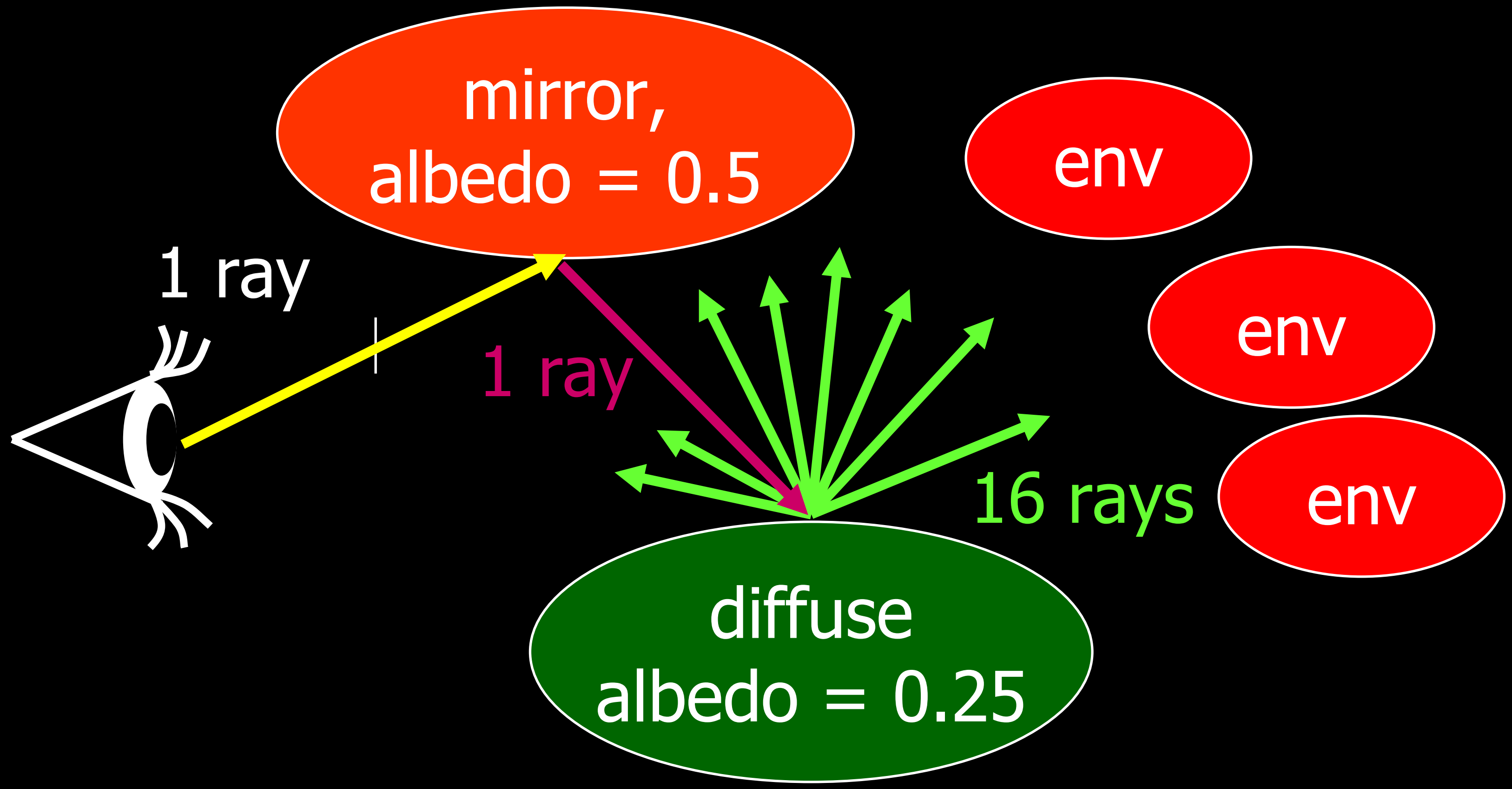
- Little reuse
- Number of samples of n-bounce is proportional to the total contribution of n-bounce paths

Example: Russian roulette



Number of samples of n-bounce is proportional to the **contribution** of n-bounce paths

Example: Go with the winners

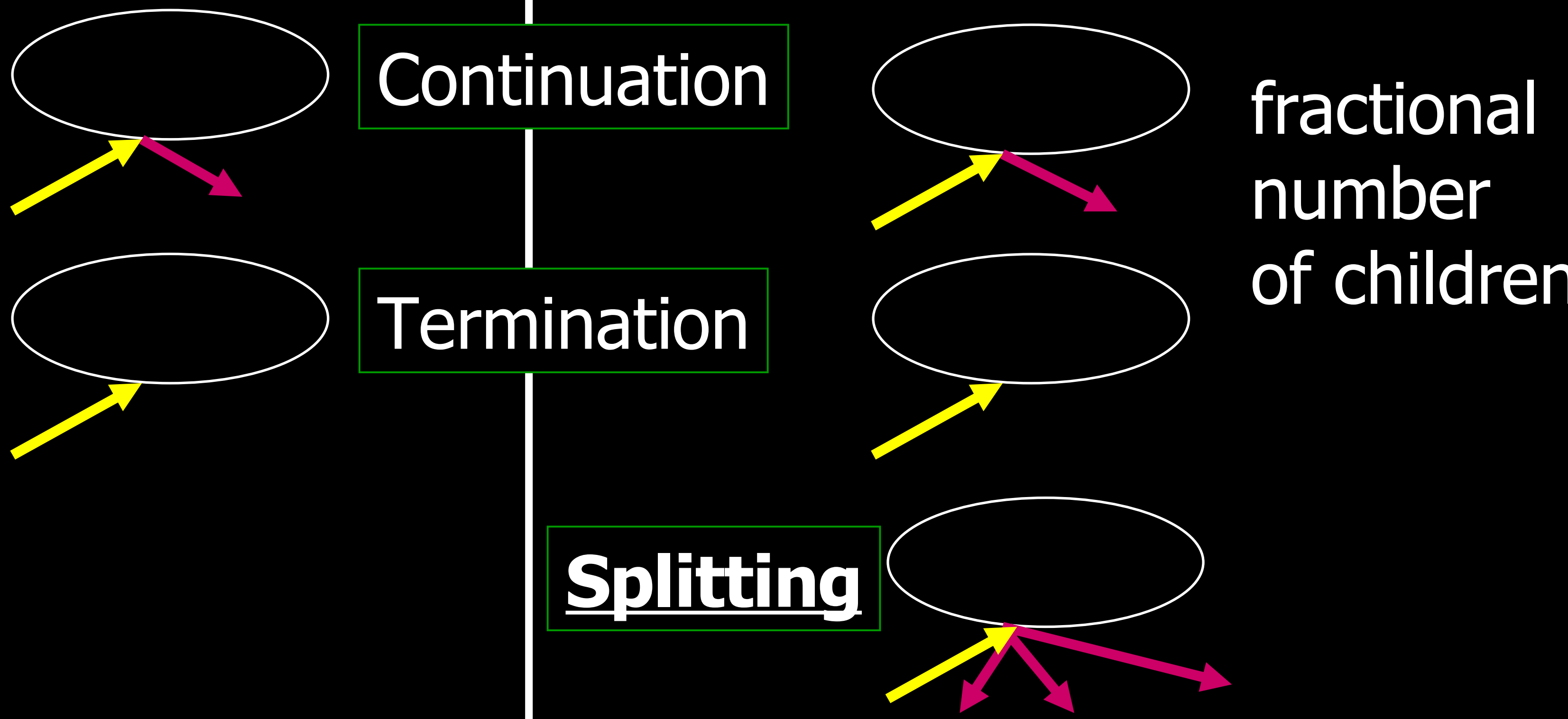


Number of samples of n-bounce is proportional to the **variance** of n-bounce paths

Random path continuation

Russian-roulette

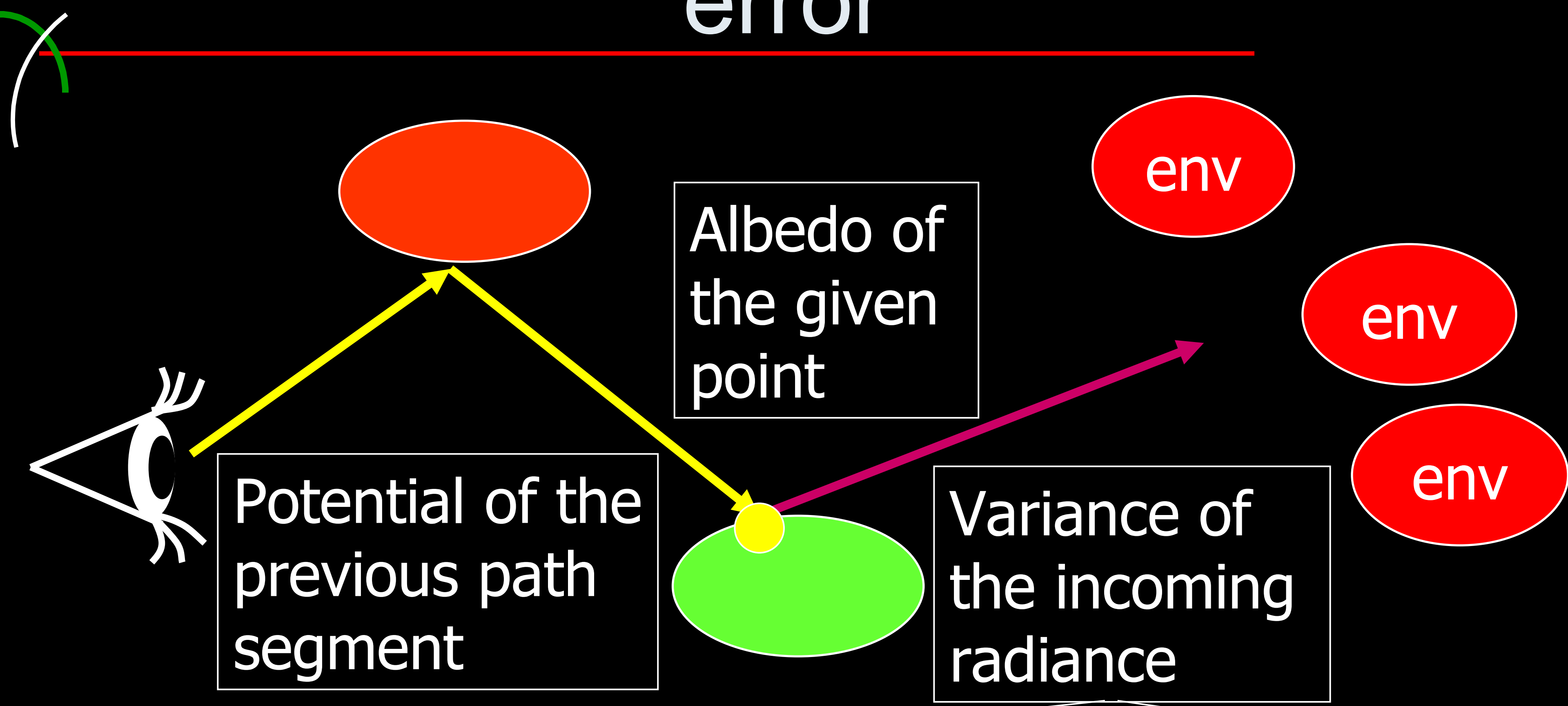
Go with the winners



Continuation with the probability of the albedo

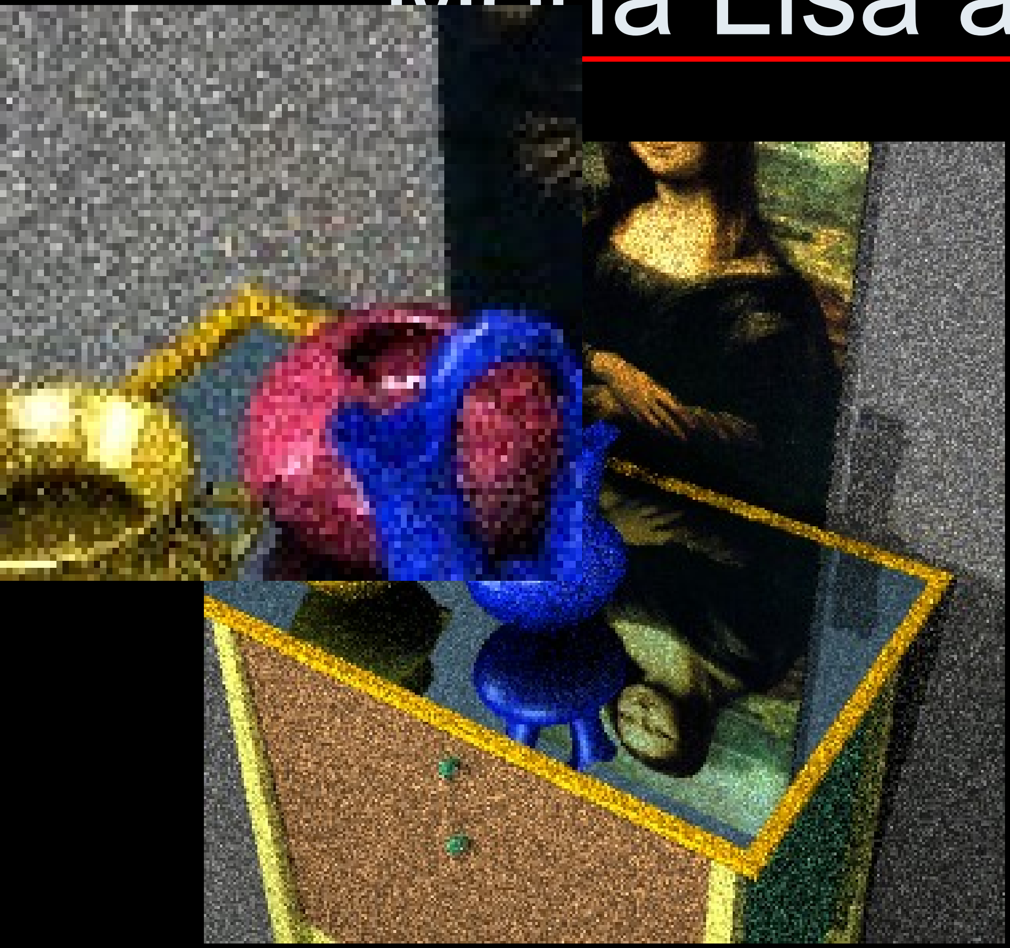
Number of children is proportional to the anticipated error

Estimation of the anticipated error

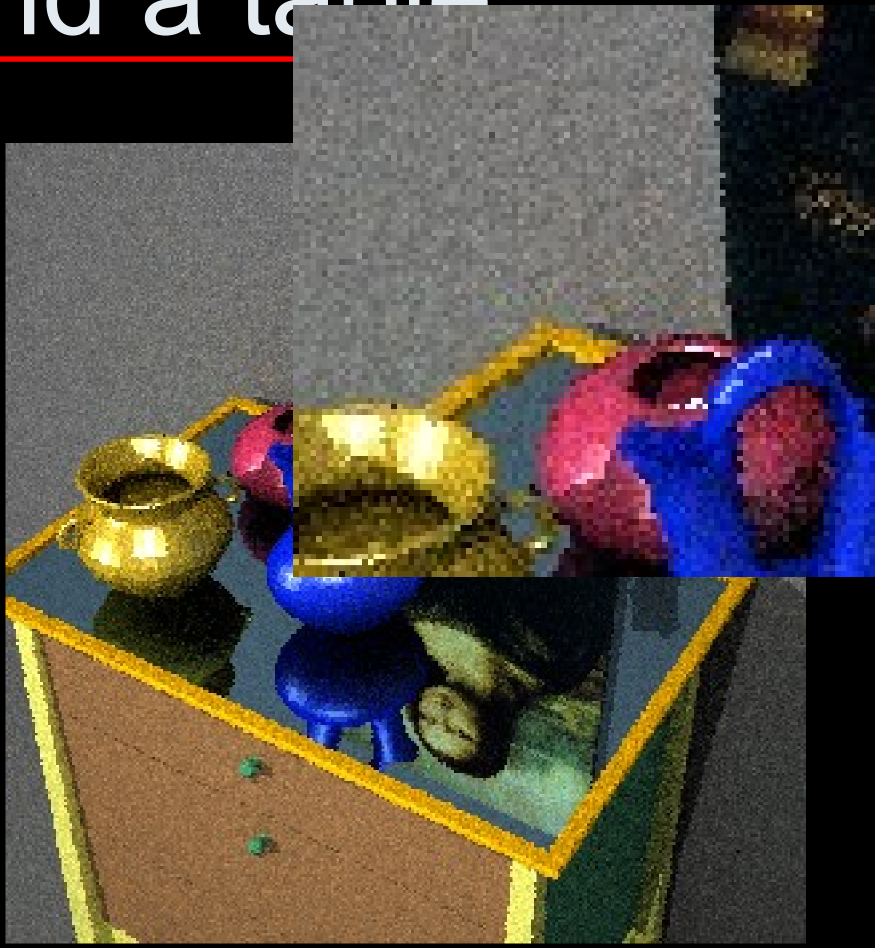


$$\frac{A}{(s+1)^2}$$ Variance of the direction (scattering) + Variance of the radiance estimation at a given direction B

Simulation results: Mona Lisa and a table



Russian roulette
2 million rays, 19 seconds



Go with the winners
2 million rays, 15 seconds

Simulation results: Mona Lisa and a tabl

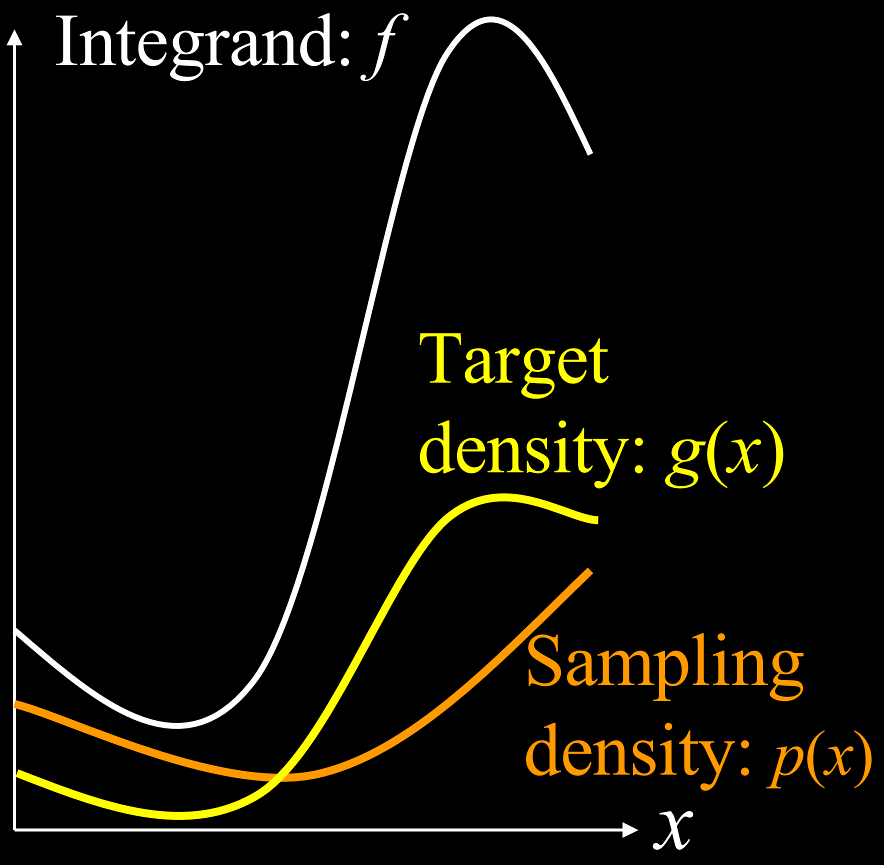
Russian-roulette
10 million rays, 104 secs

Go with the winners
10 million rays, 76 secs

Example 4

Improved Indirect Photon Mapping
with
Weighted Importance Sampling

Weighted Importance Sampling



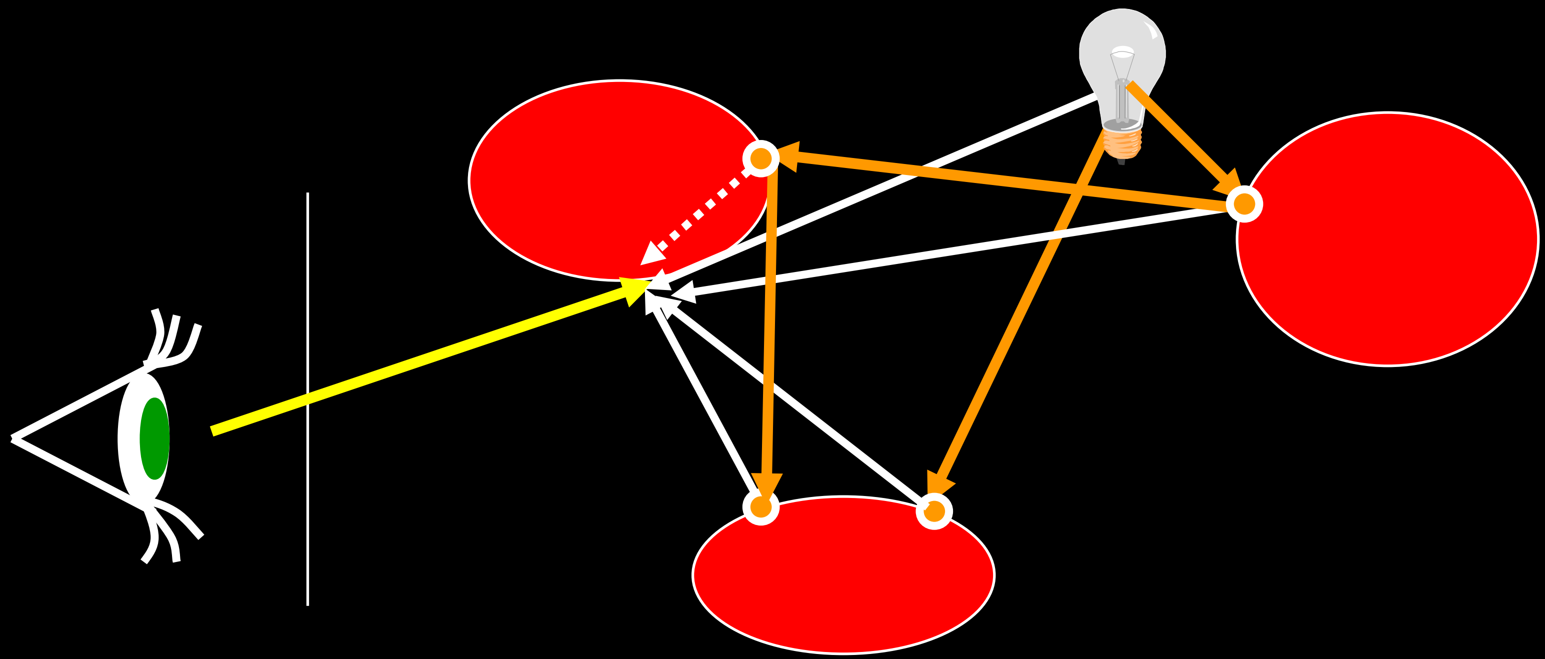
Classical Monte Carlo Estimate:

$$\frac{\Sigma f(x_i)/p(x_i)}{M}$$

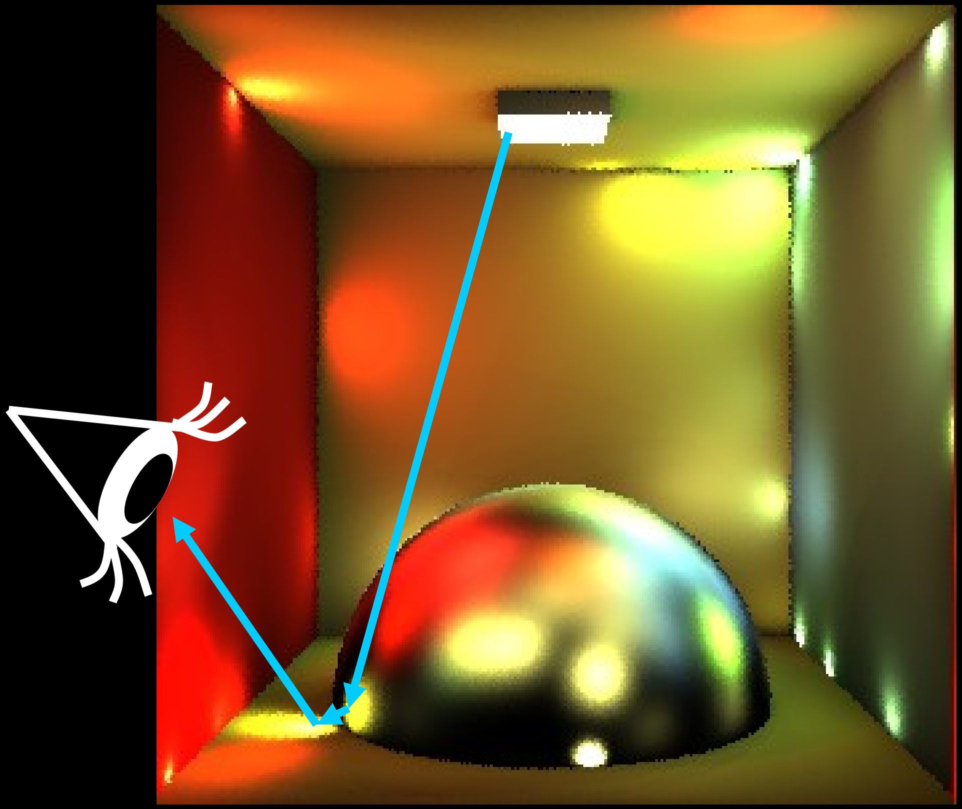
Weighted Monte Carlo Estimate:

$$\frac{\Sigma f(x_i)/p(x_i)}{\Sigma g(x_i)/p(x_i)}$$

Virtual light sources (instant radiosity, indirect photon mapping)

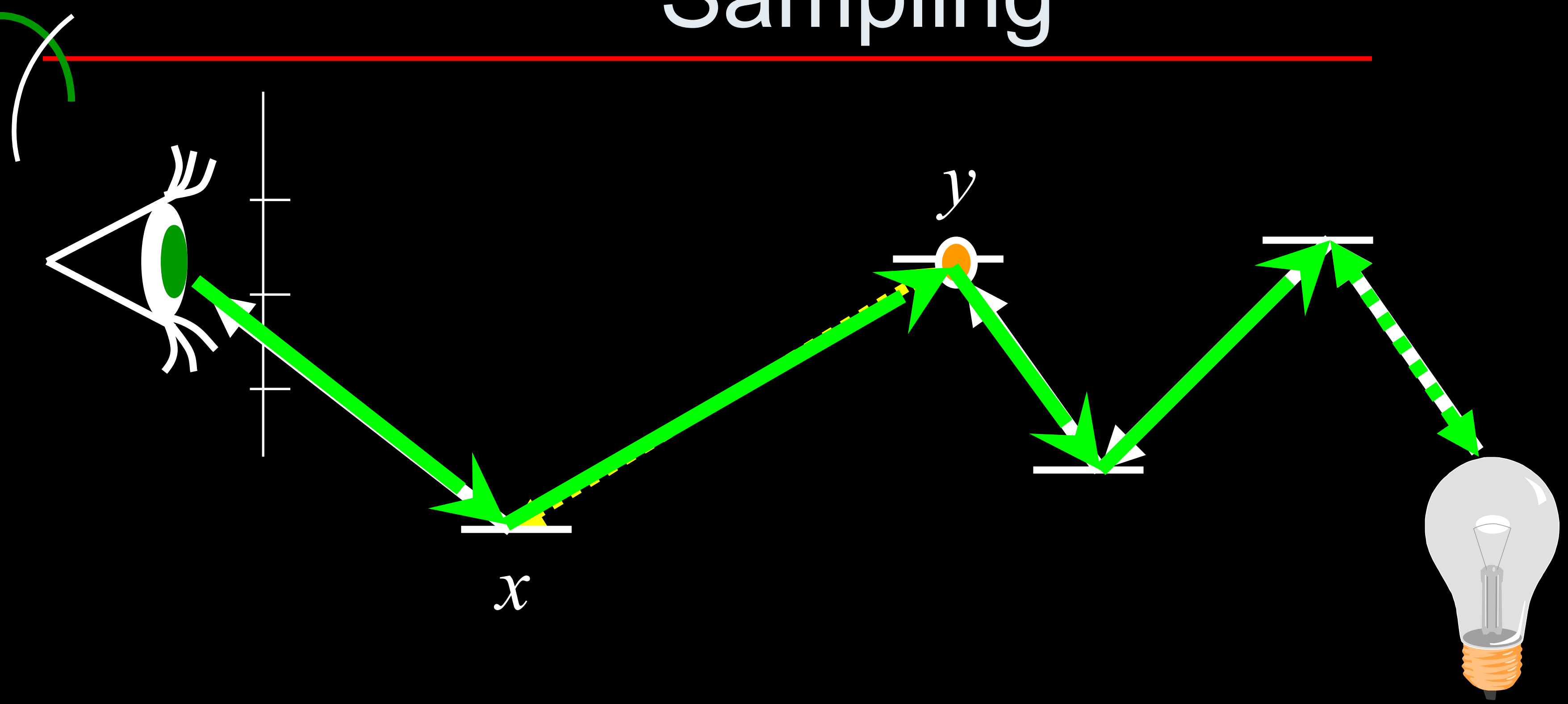


Radiance estimate for virtual light sources



High contribution sample
generated with relatively low probability

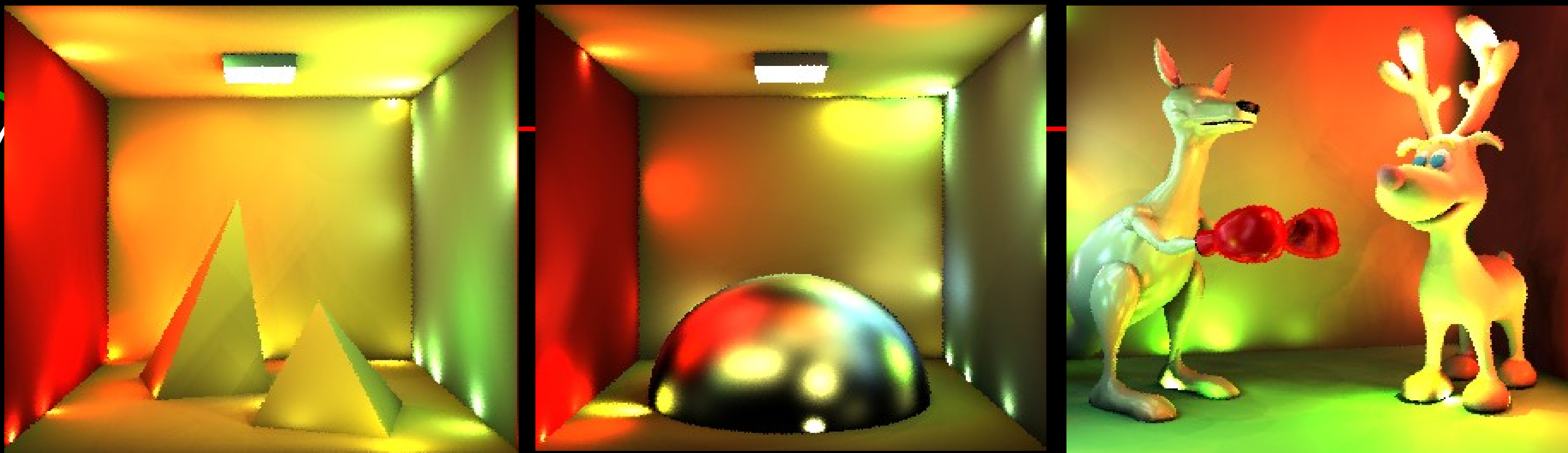
Application of Weighted Importance Sampling



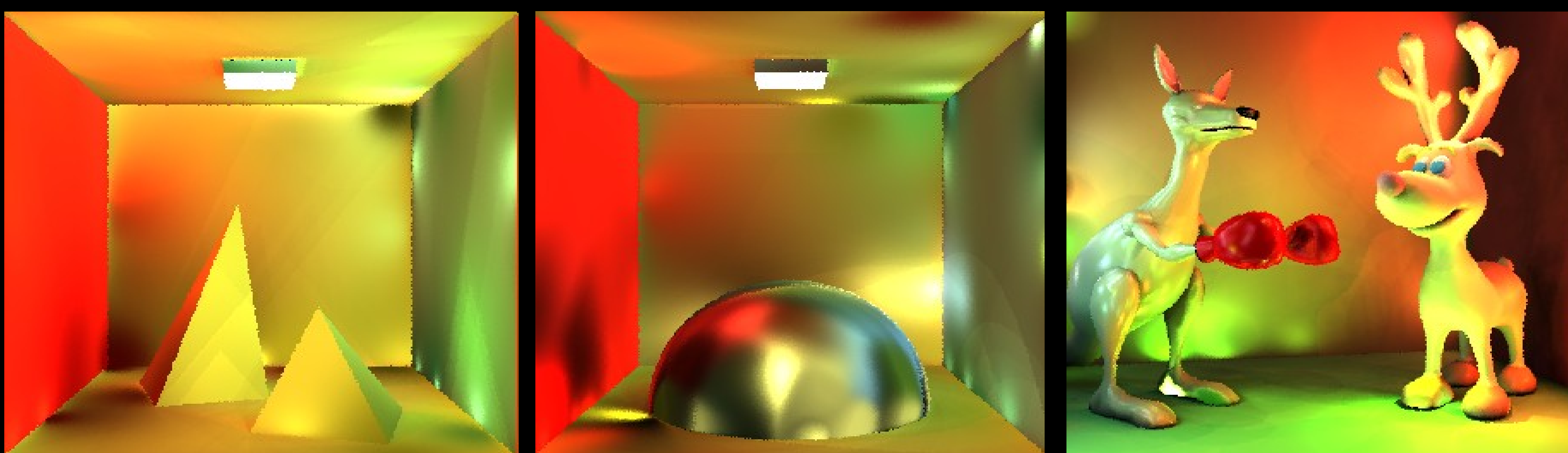
Target g:

the probability density of path tracing

Original indirect photon mapping (no direct illumination)



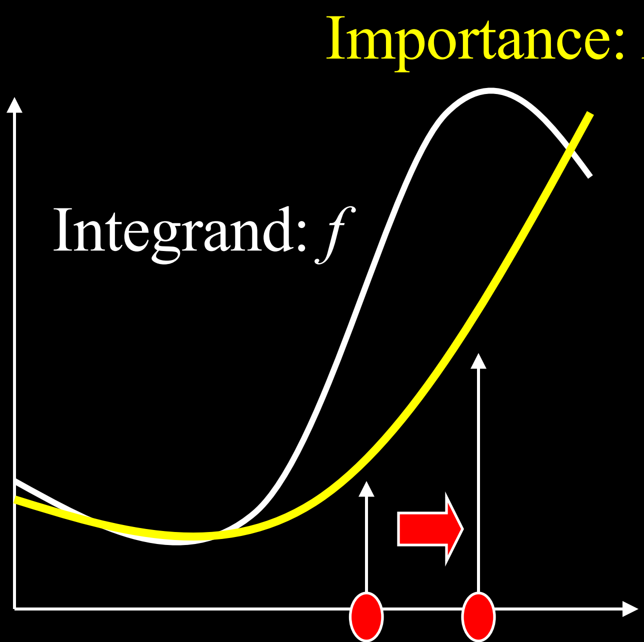
With weighted importance sampling (no direct illumination)



Example 5

A Simple and Robust Mutation Strategy for the Metropolis Light Transport

Metropolis Sampling



$$a(x \to y) = \frac{I(y) \cdot T(y \to x)}{I(x) \cdot T(x \to y)}$$

1. Find I that mimics f
2. Find the normalization constant: $b = \int I \, dx$

Sampling:

Mutation/Acceptance

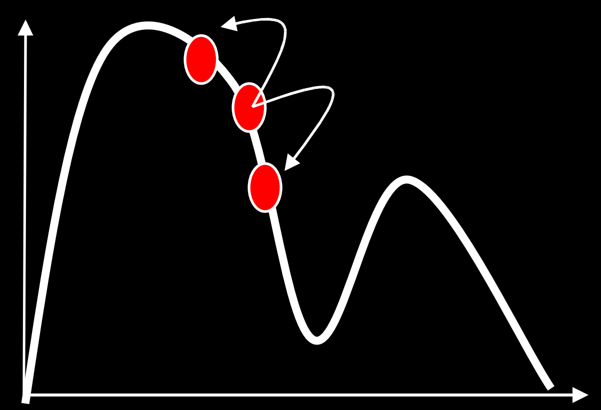
- arbitrary mutation $T(x \to y)$
- carefully selected acceptance probability $a(x \to y)$

Drawbacks of Metropolis

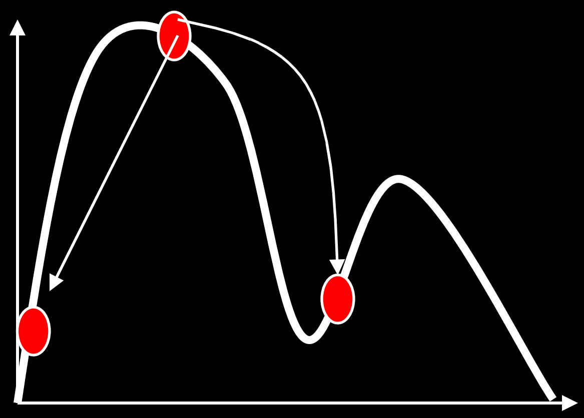
- Start-up bias
  - Process only converges to the stationary state
- Correlated samples
  - Increase the variance of the integral quadrature
- Number of samples in a pixel $\propto I$
  - few samples for dark regions

Good mutation strategy

- Quickly forgets previous samples
- Reduces the correlation of samples



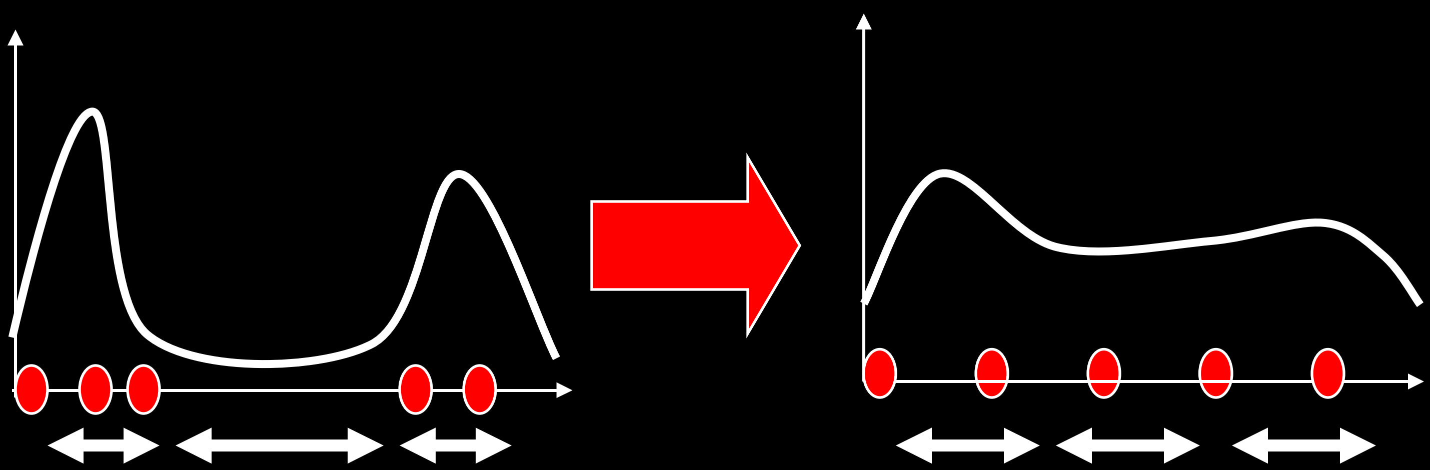
Small mutations are bad



Large mutations can also be bad around the peaks

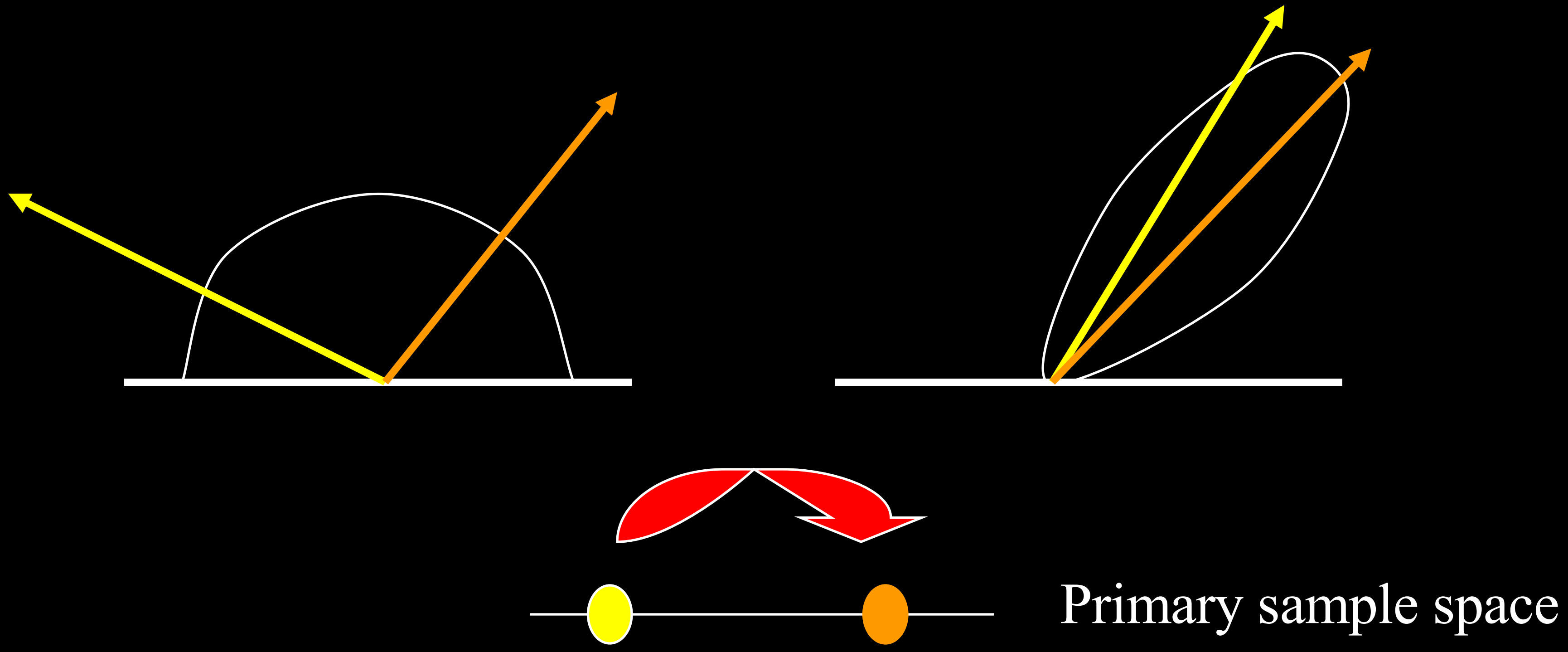
Importance controlled mutation size

- Big mutations at unimportant regions and fine, small mutations at important regions
- Transform the domain to expand important regions and shrink unimportant regions and use uniform perturbations

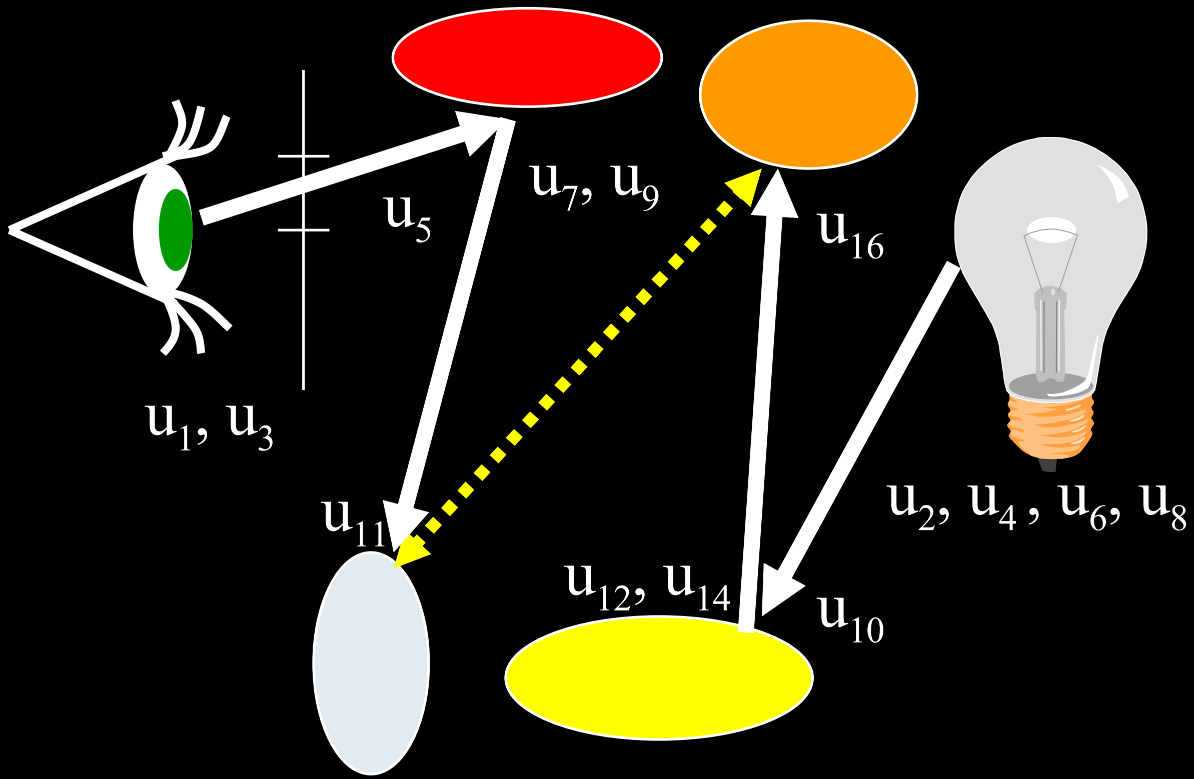


Perturbing in the space of pseudo-random numbers

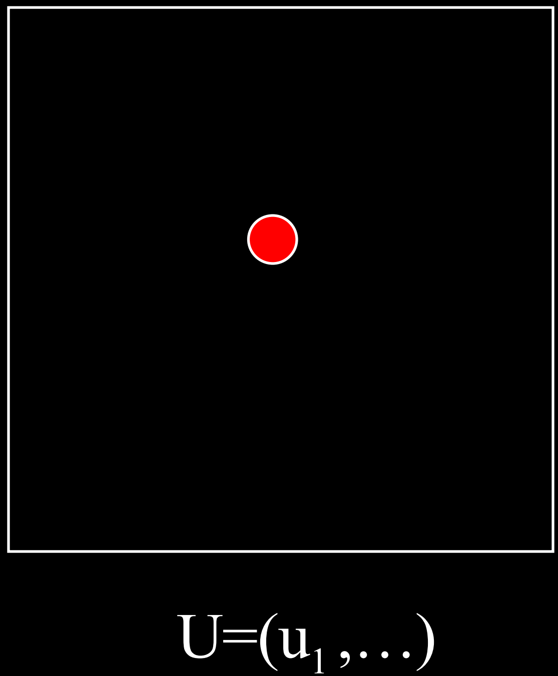
- Transformation **for free**: BRDF sampling, lightsource sampling, Russian Roulette



Mutating in the Primary Sample Space

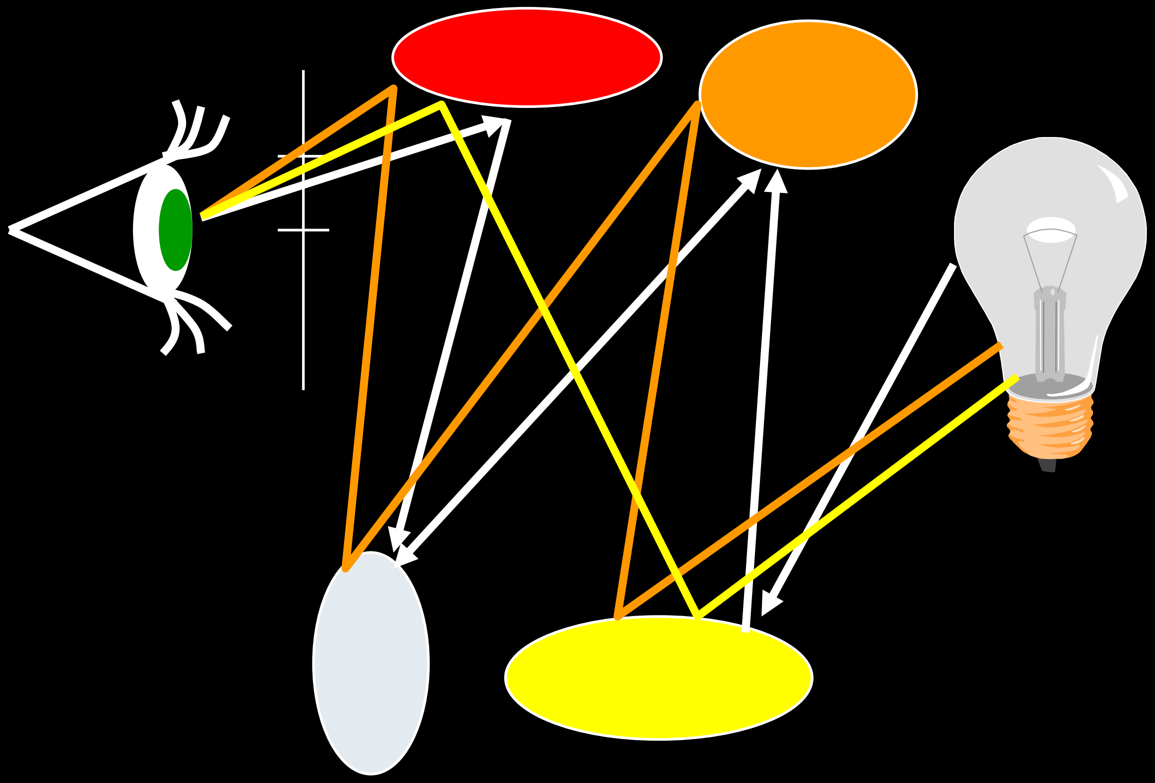


Path space

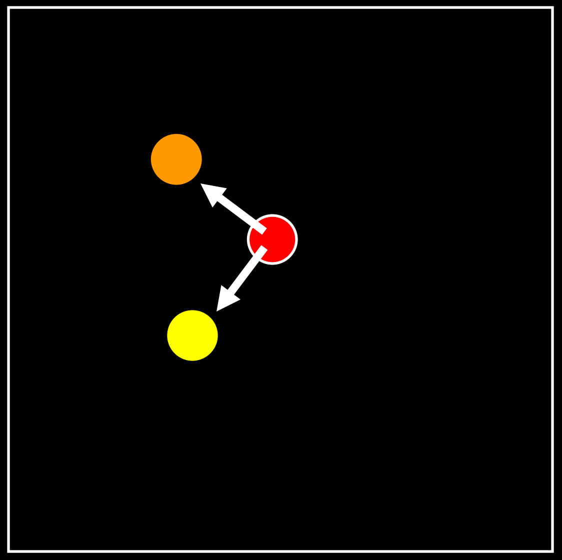


Primary sample space

Mutating in the Primary Sample Space



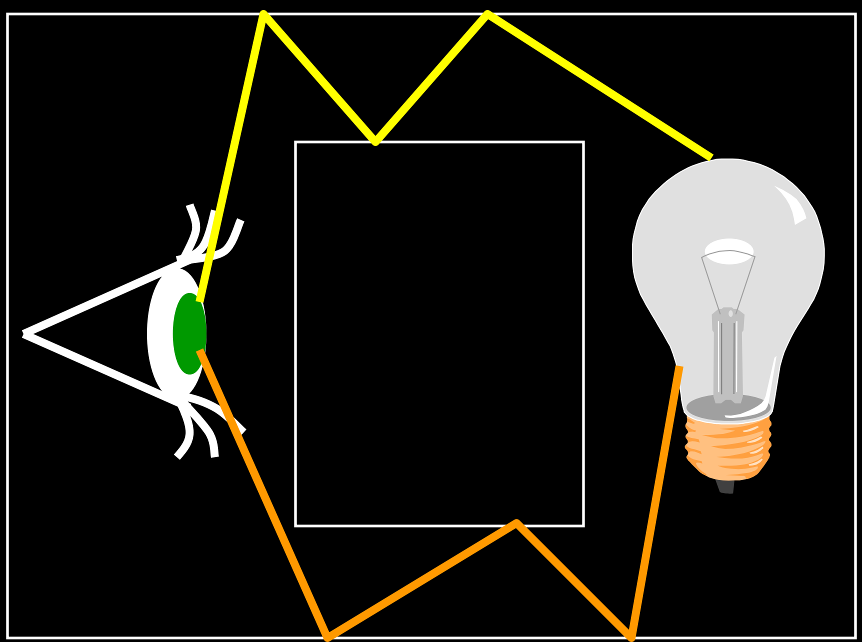
Path space

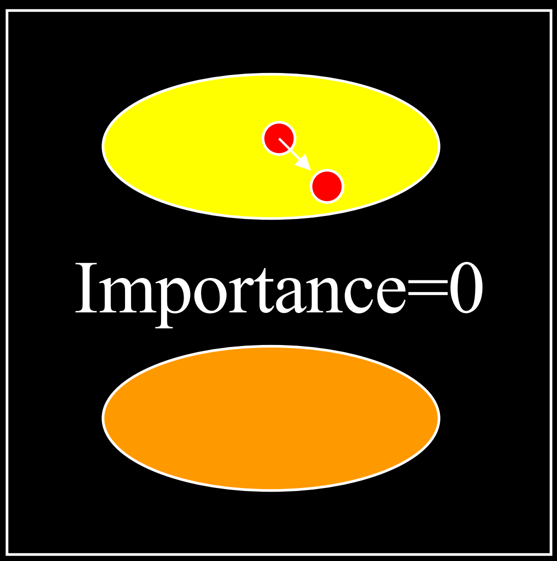


$U=(u_1,\ldots)$

Primary sample space

Ergodicity: Large (independent) Steps



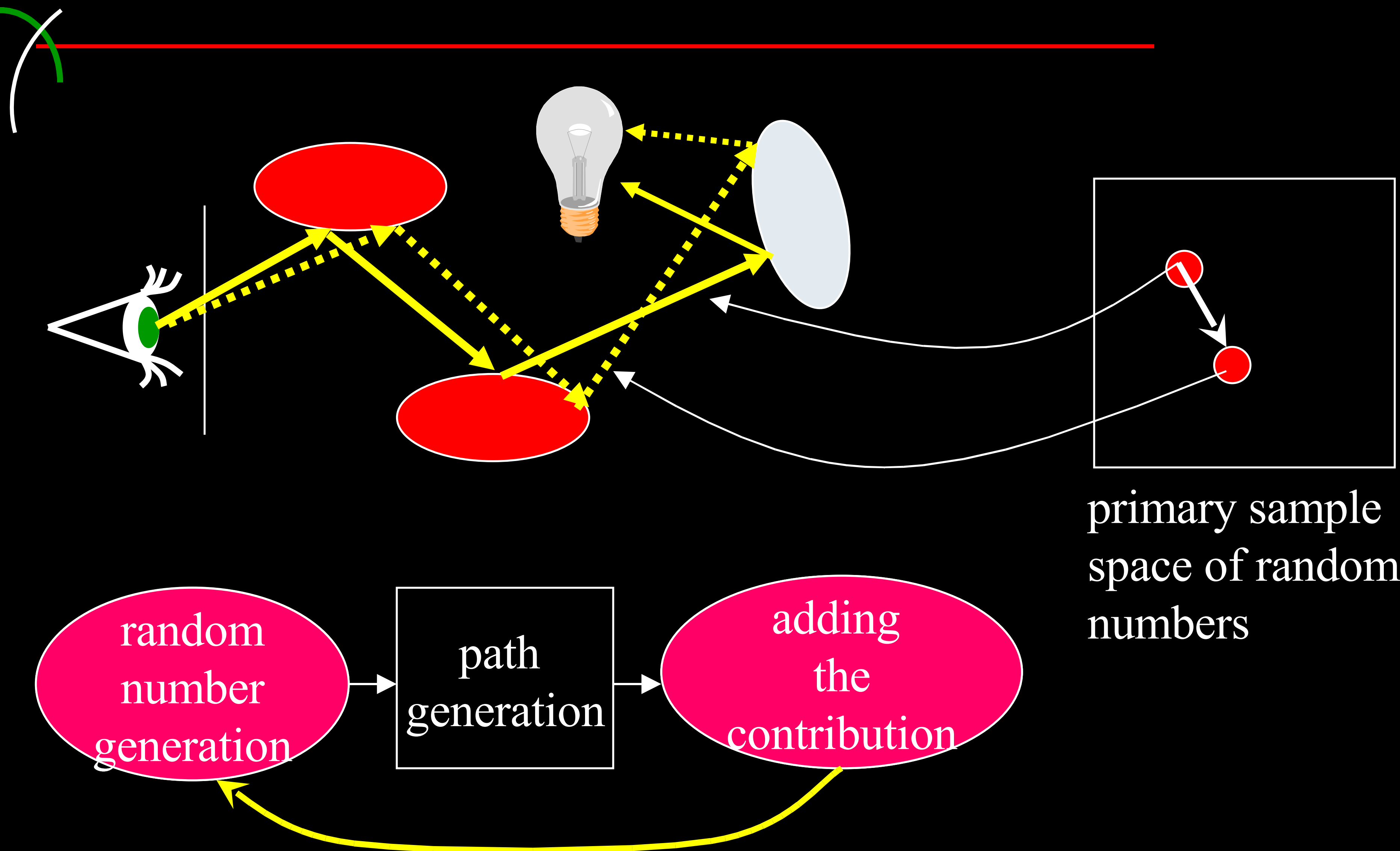


1. Small steps with peturbation
2. Large steps independently of the actual sample: p_{large}

Benefits of Large steps

- Ergodicity
- Sampling process forgets
- Reduces the start-up bias
- Can be used to compute the normalization constant *b*
- Sequence of large steps is a conventional random walk: Combination with Metropolis
  - multiple importance sampling

Implementation



Bidir path tracing

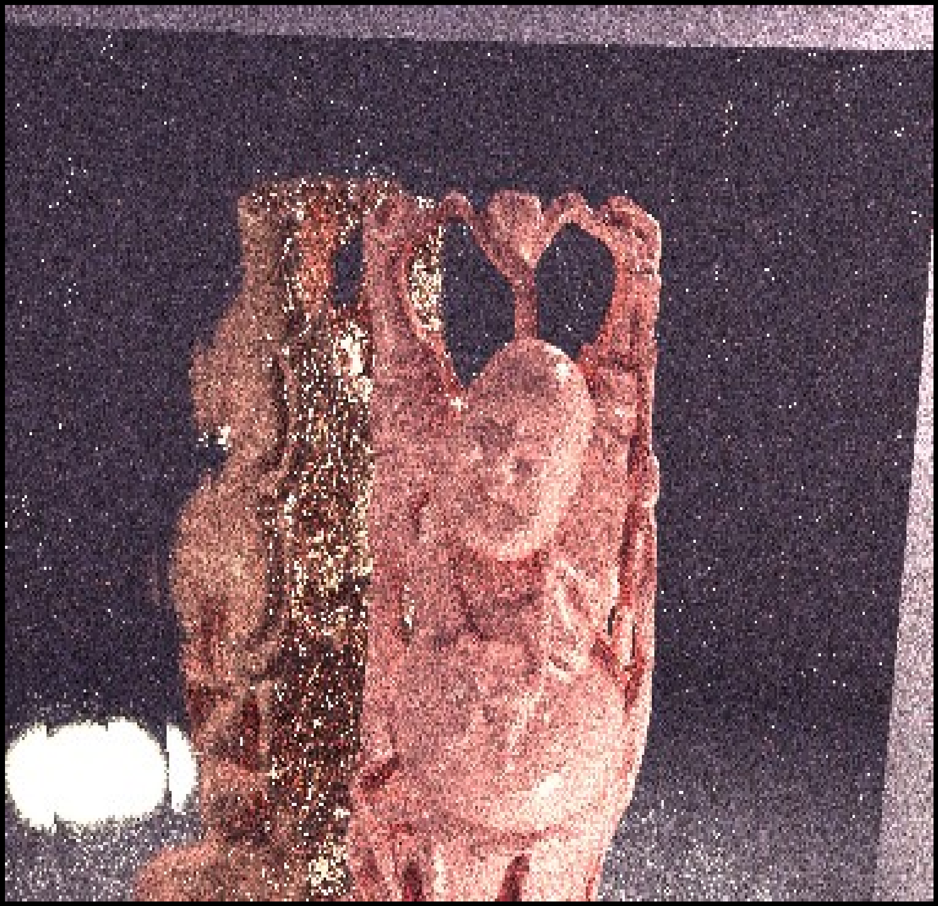
Metropolis

25 samples per pixel

Effects of large step probability

p_{large}=0.02 p_{large}=0.5 p_{large}=0.9

Multiple Importance sampling



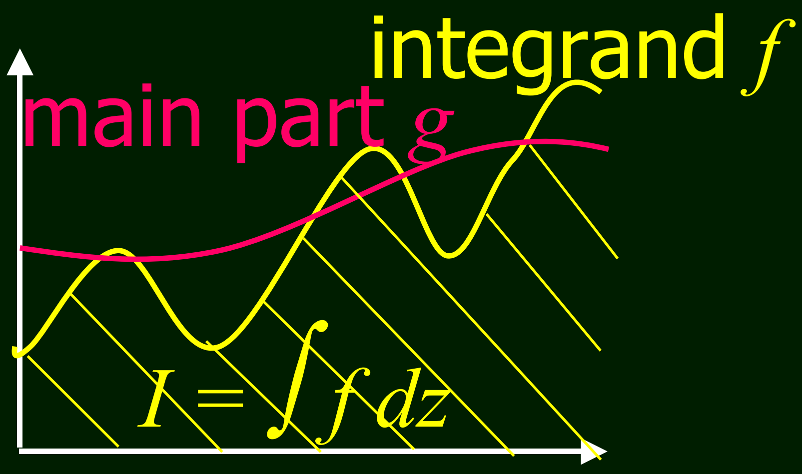
Mean value substitution

Multiple Importance sampling

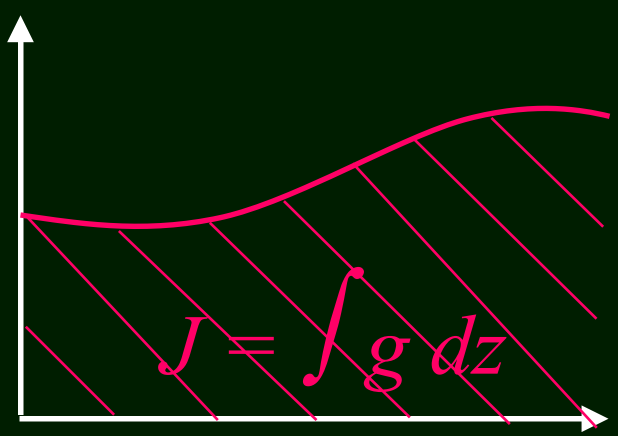
Example 6

Combined Correlated and
Importance Sampling
in Direct Illumintion Computation
for Area Lights
and Environment Mapping

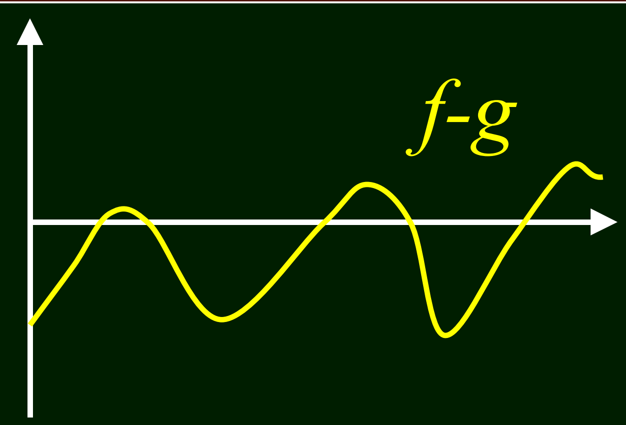
Correlated sampling



$$I = \int f \, dz = \int g \, dz + \int f\text{-}g \, dz = J + \int f\text{-}g \, dz$$



+



Problem spots - example

direct lighting, area light source

could be calculated analytically

↓

correlated sampling

Problem spots - example

direct lighting, area light source

could be calculated analytically

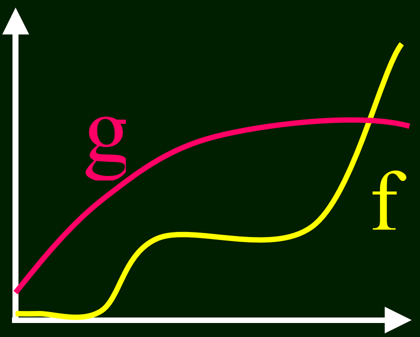
correlated sampling

Linear combination

correlated estimator

$f \cong g$

$$J + \left(\frac{f-g}{p}\right) = \frac{f}{p} + \left(J - \frac{g}{p}\right)$$



importance estimator

$g \neq f \cong 0$

$$\frac{f}{p}$$

$$\frac{f}{p} + \lambda\left(J - \frac{g}{p}\right)$$

Finding the λ

minimizing the variance:

$$\sigma^2(\lambda) = E\left(\left(\frac{f(z)}{p(z)} + \lambda\left(J - \frac{g(z)}{p(z)}\right) - I\right)^2\right)$$

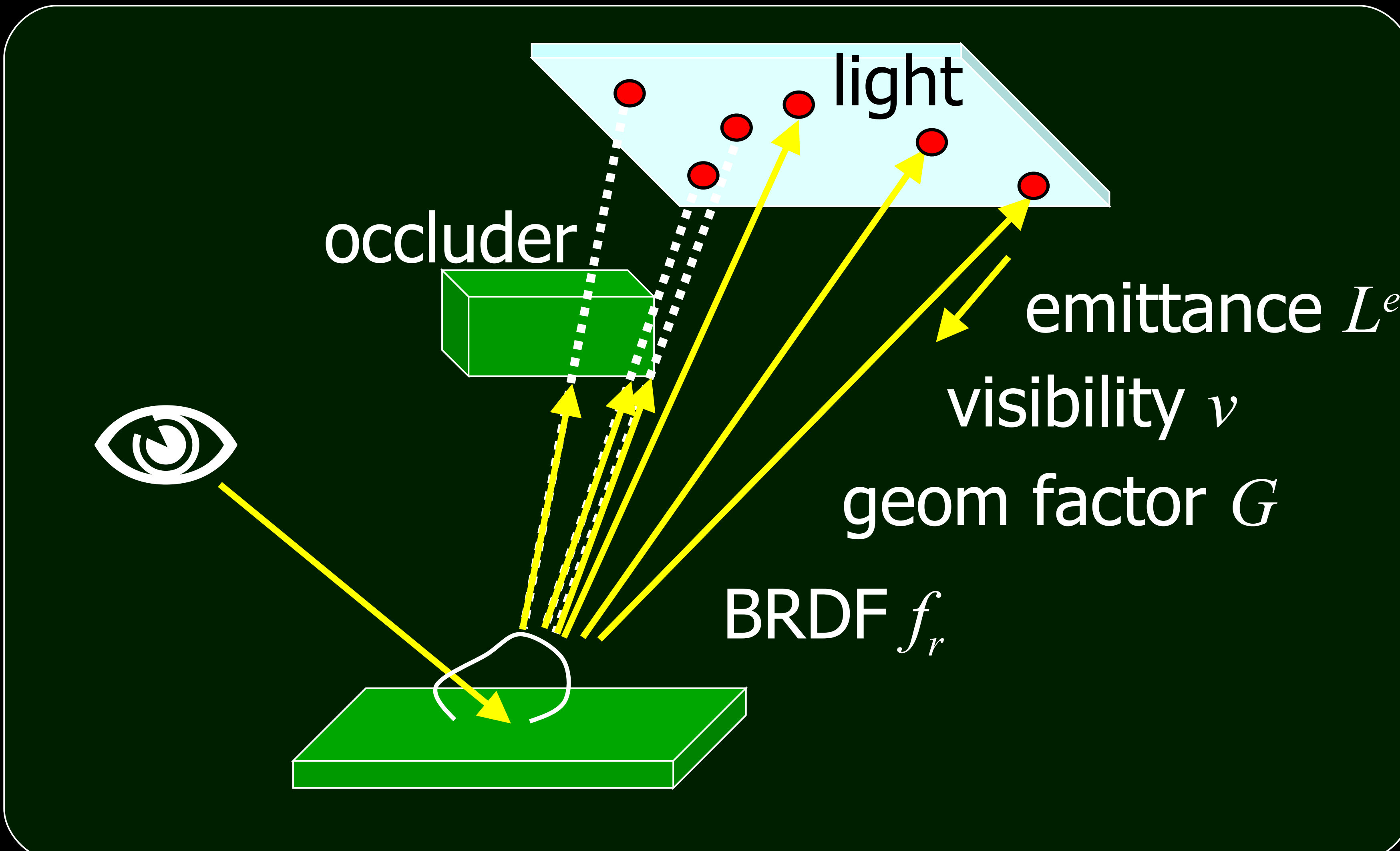
provides the formula:

$$\lambda = \frac{E\left(\left(J - \frac{g(z)}{p(z)}\right)\left(I - \frac{f(z)}{p(z)}\right)\right)}{E\left(\left(J - \frac{g(z)}{p(z)}\right)^2\right)}$$

λ ~ correlation of f/p and g/p

computed from the samples → only asymptotically unbiased

Light source sampling



Main part

- no occlusion $v \rightarrow 1$
- uniform emittance $L^e \rightarrow \widetilde{L}^e$
- diffuse surface $f_r \rightarrow \widetilde{f}_r$

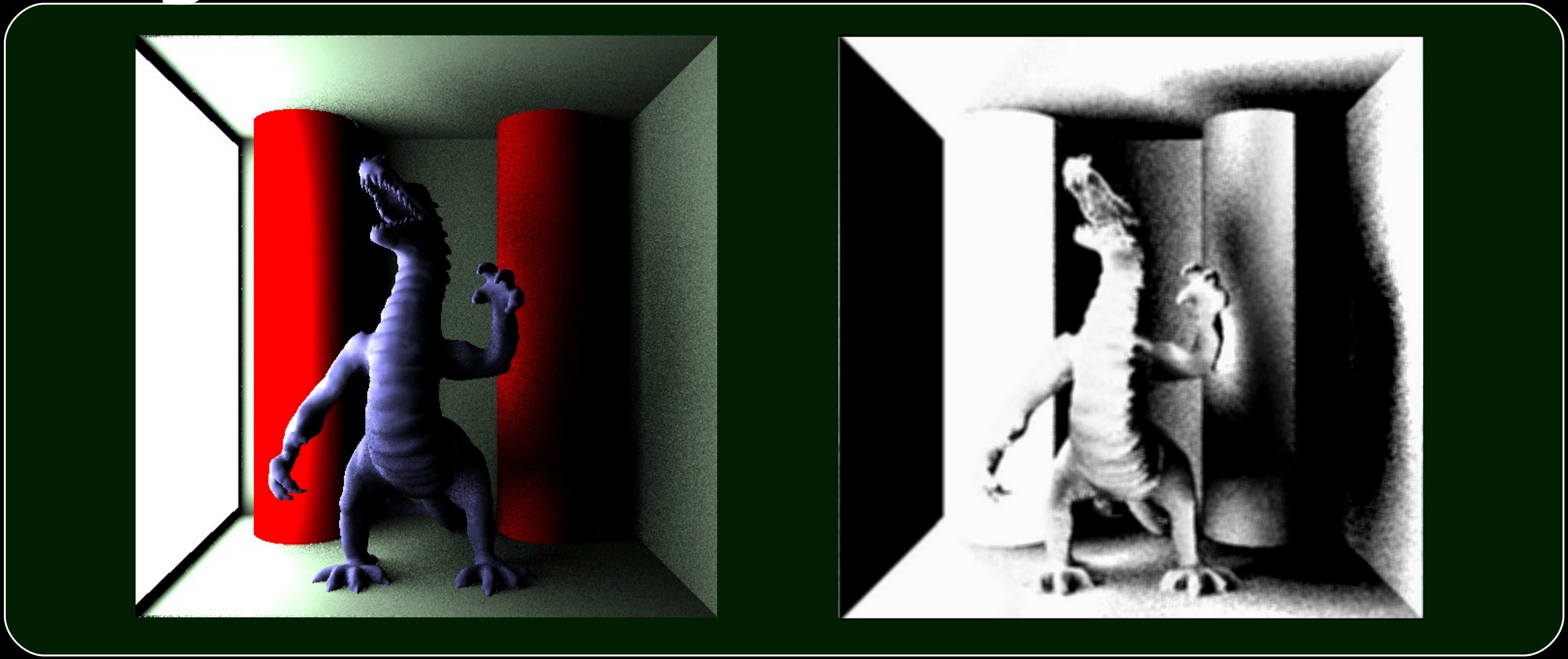
$$g = \widetilde{L}^e \cdot \widetilde{f}_r \cdot G$$

$$J = \int g = \widetilde{L}^e \cdot \widetilde{f}_r \cdot \int G$$

point-to-polygon form factor

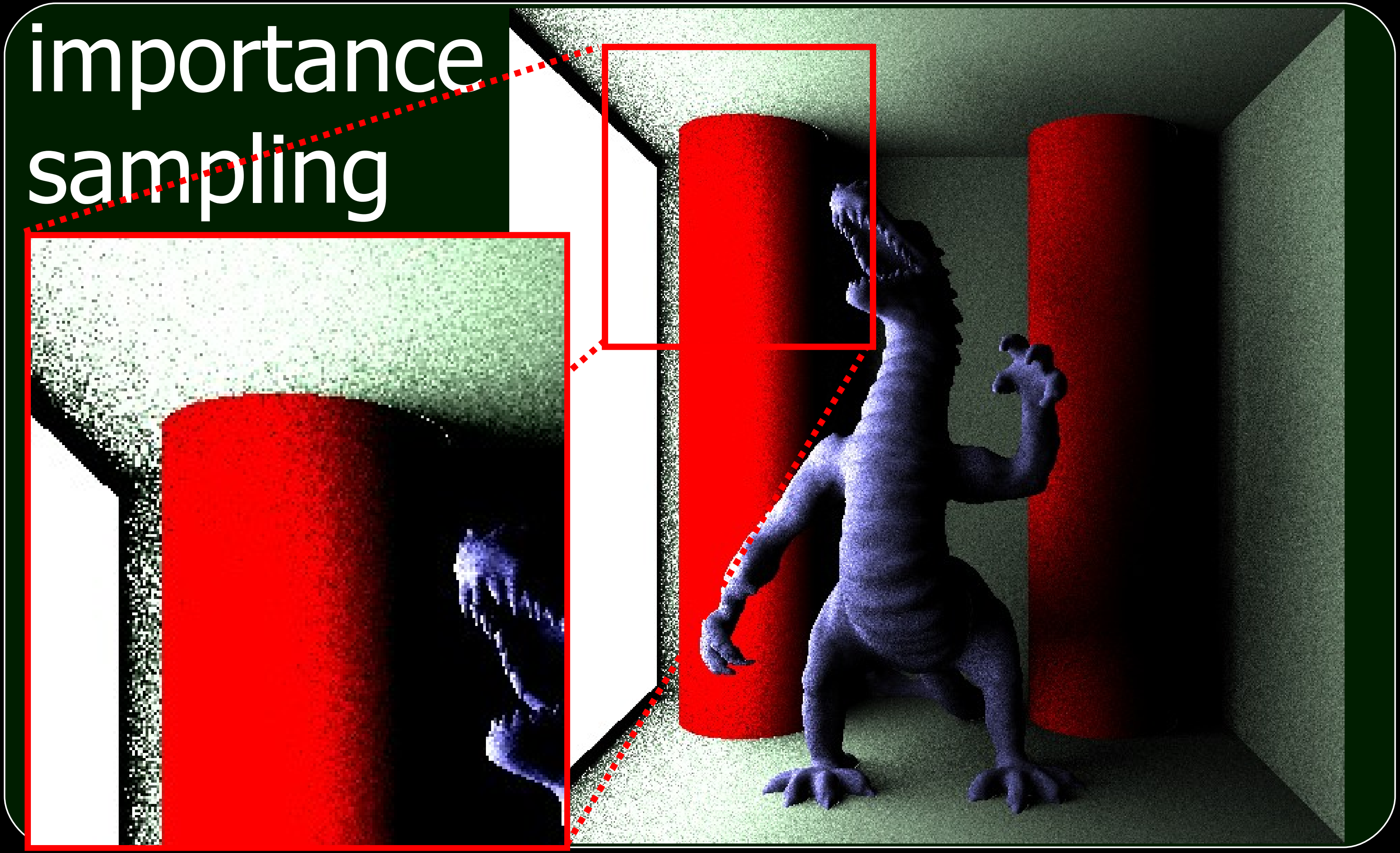
λ calculation

using derived formula

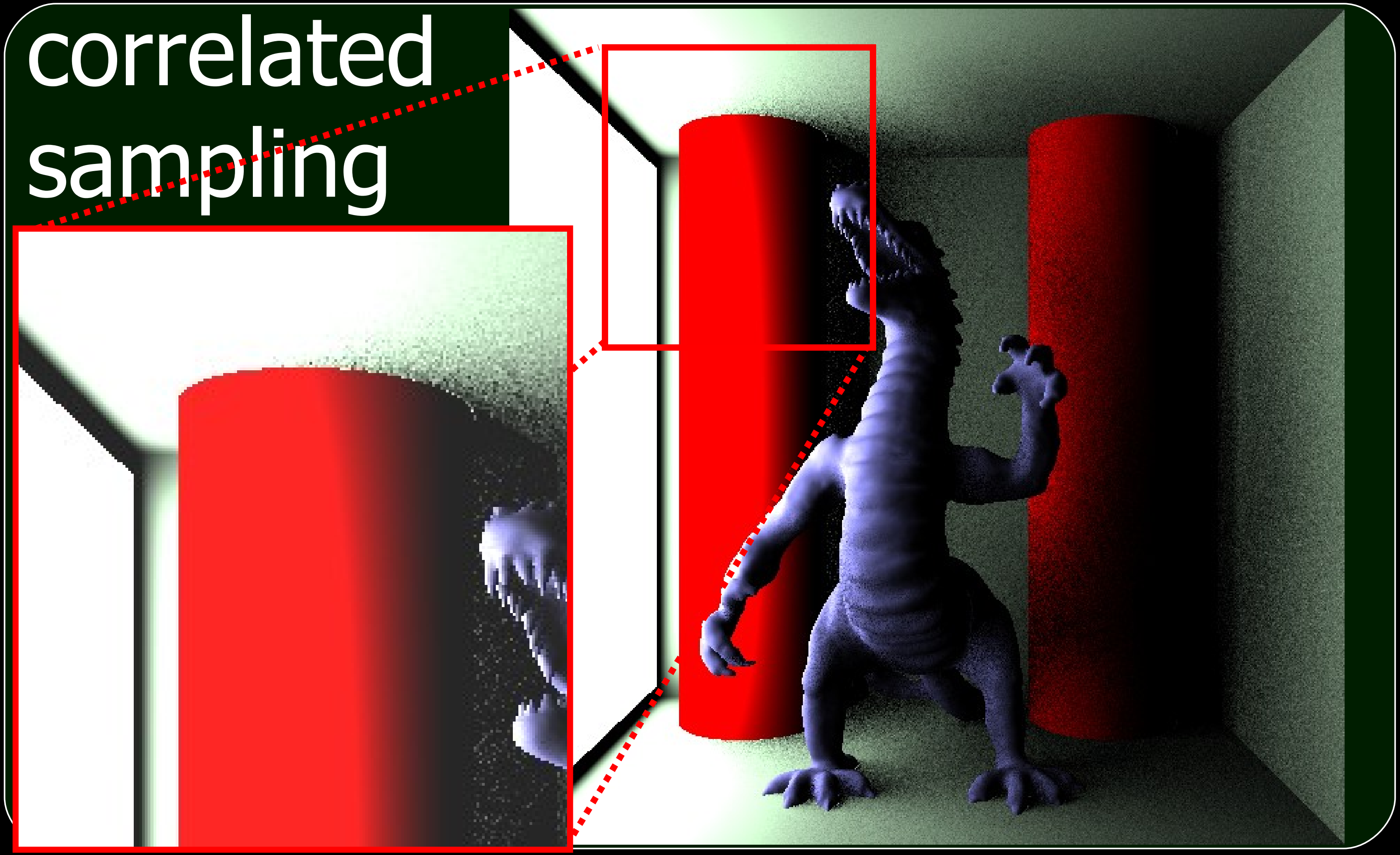


$\lambda = 1$ if fully visible
$\lambda = 0$ if fully occluded
} fractional visibility

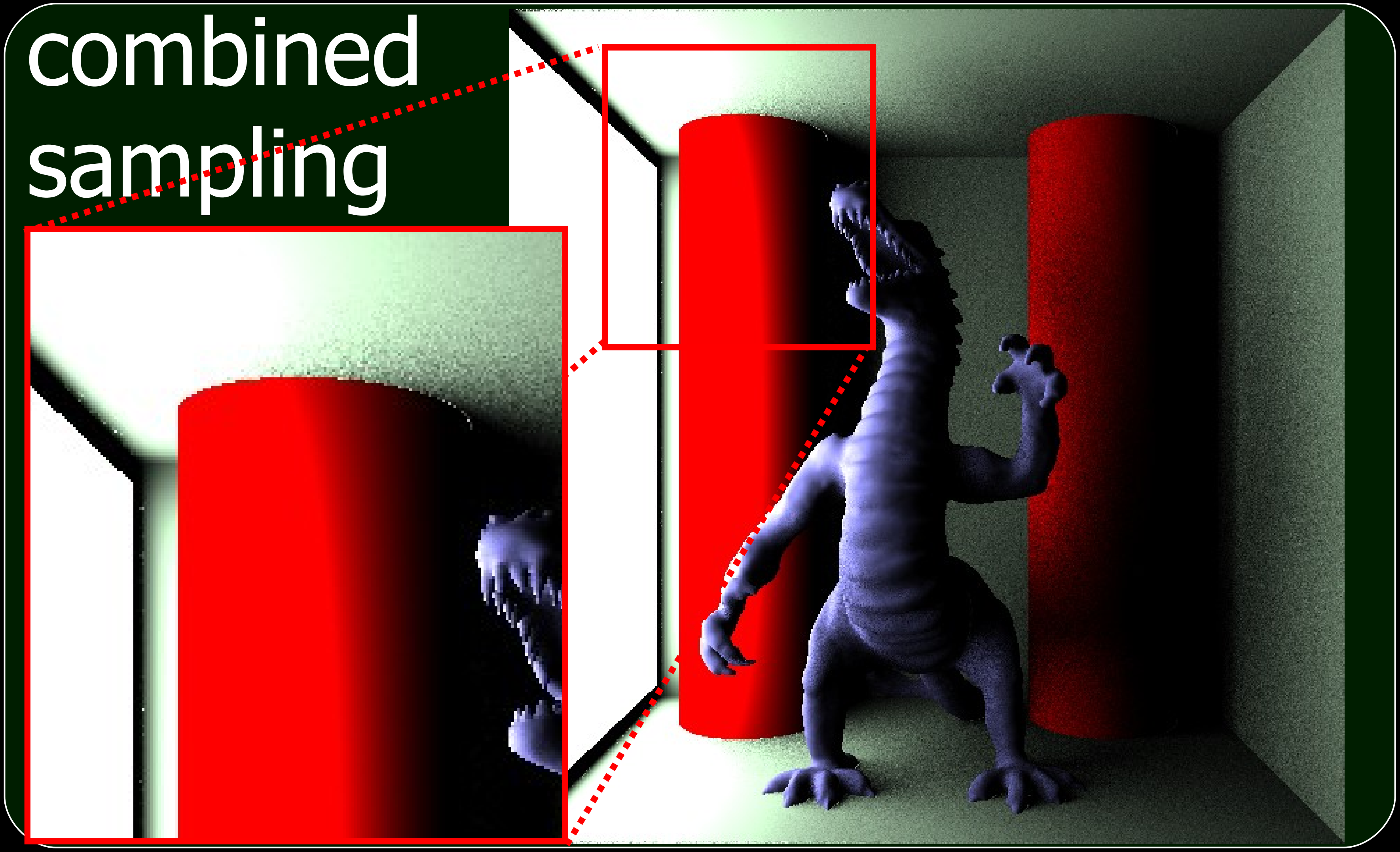
Results - images



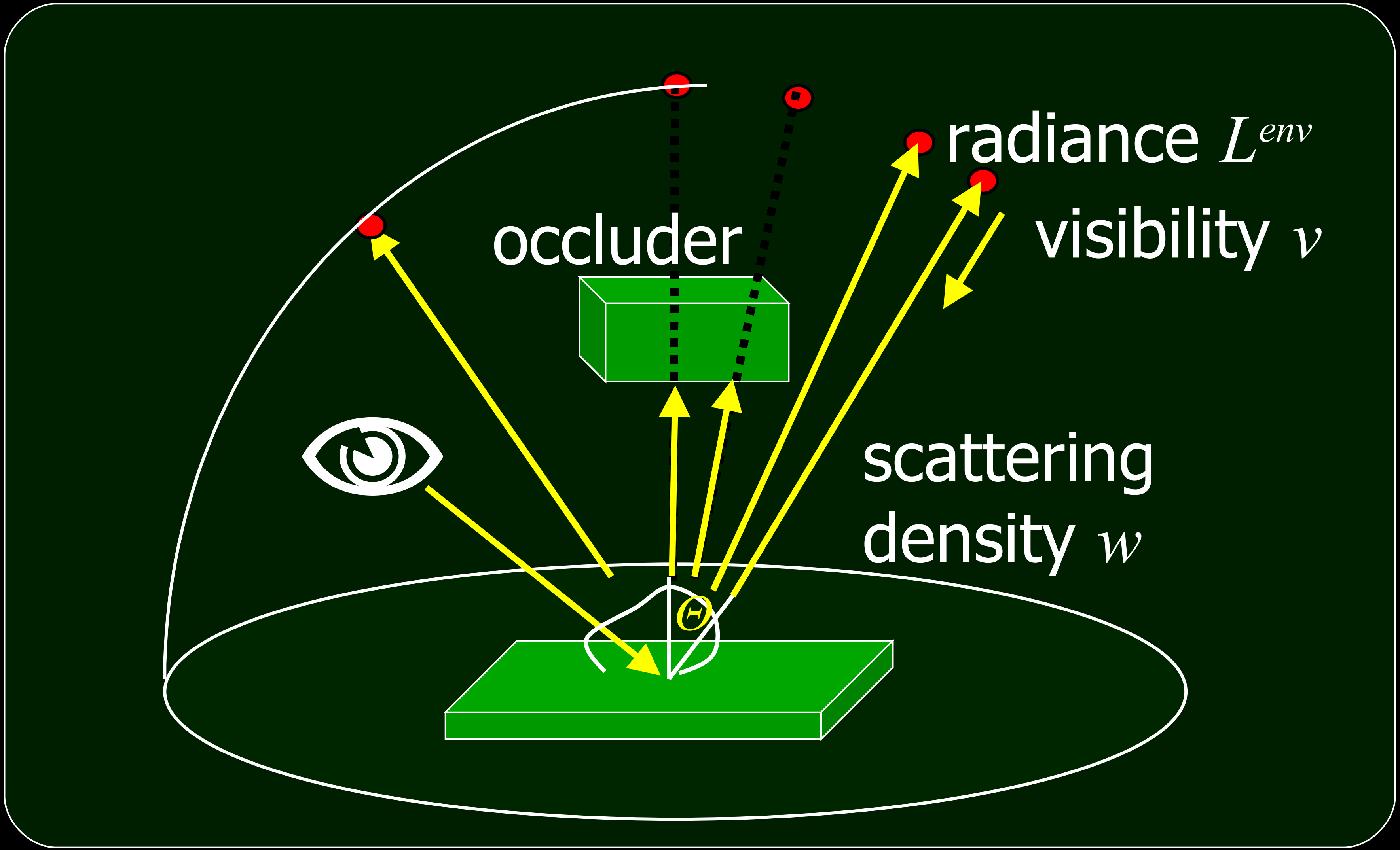
Results - images



Results - images



Environment mapping & skylight illumination



Main part

$$f = L^{env} \cdot v \cdot w$$

$\widetilde{L}^{env} = \int L^{env}$

no occlusion

$a_d = \int w$

$$g = \widetilde{L}^{env} \cdot w$$

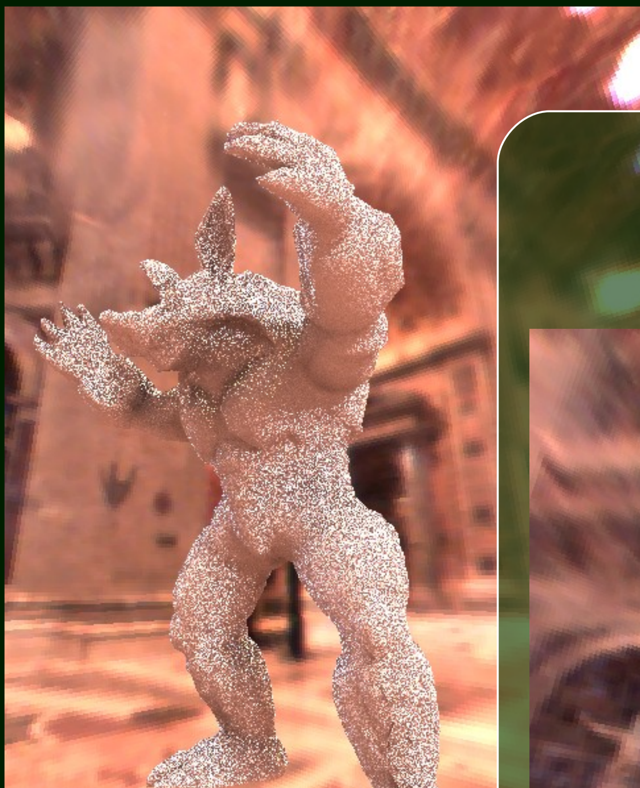
$$g = L^{env} \cdot a_d$$

smooth map

non-specular

$$J = \int g = \widetilde{L}^{env} \cdot a_d$$

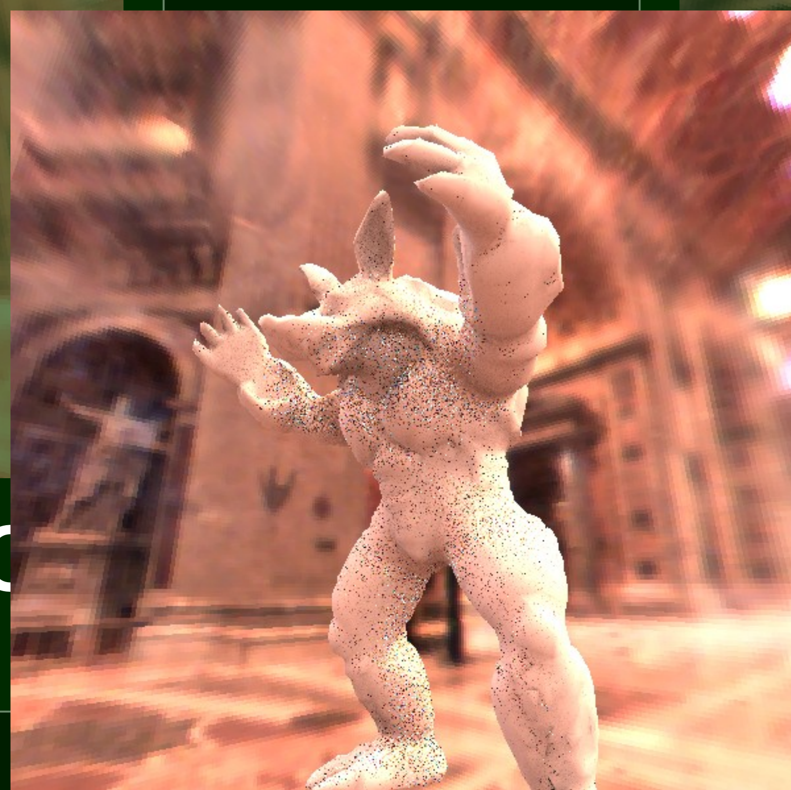
Environment mapping results



importanc

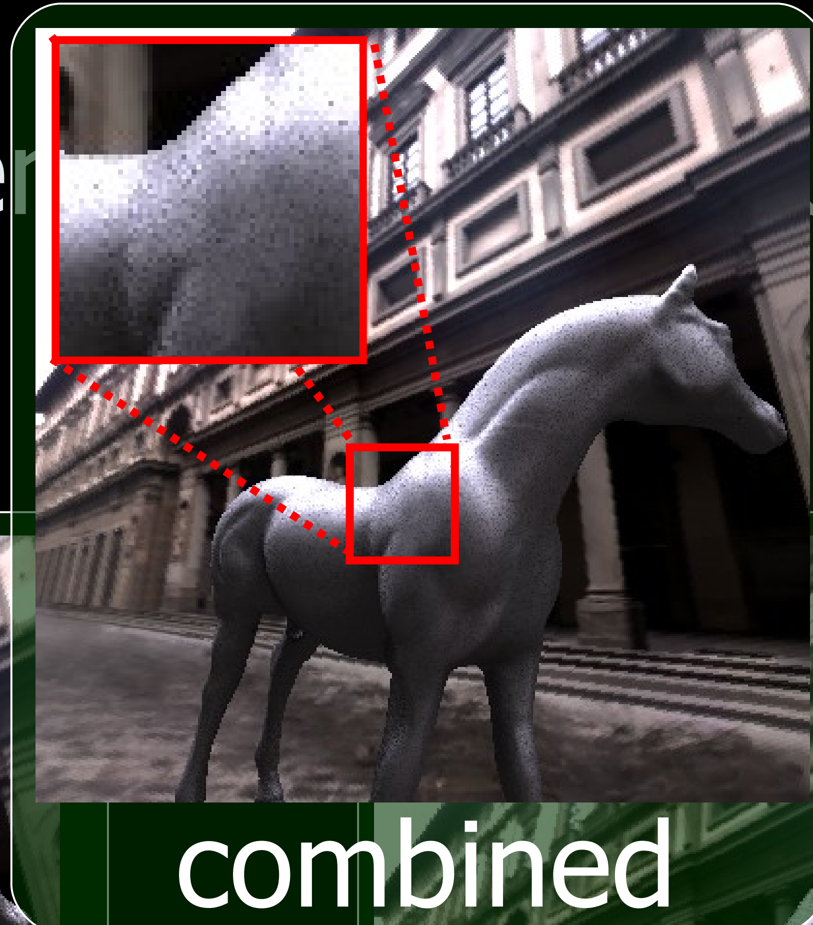
(emittance)

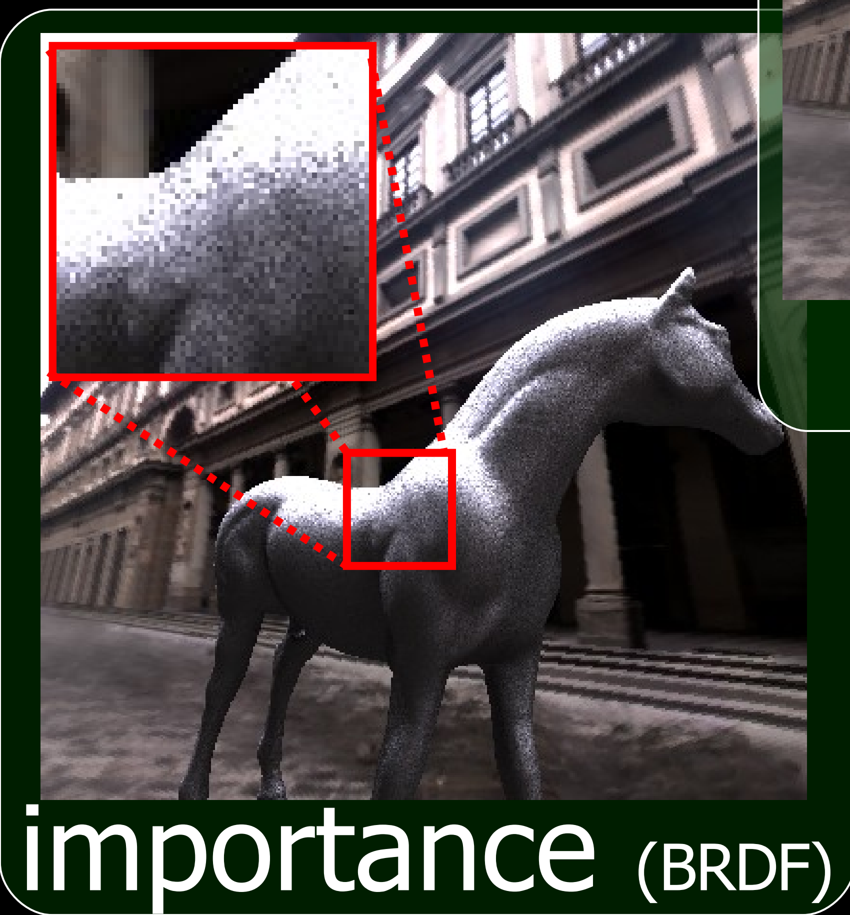
combined

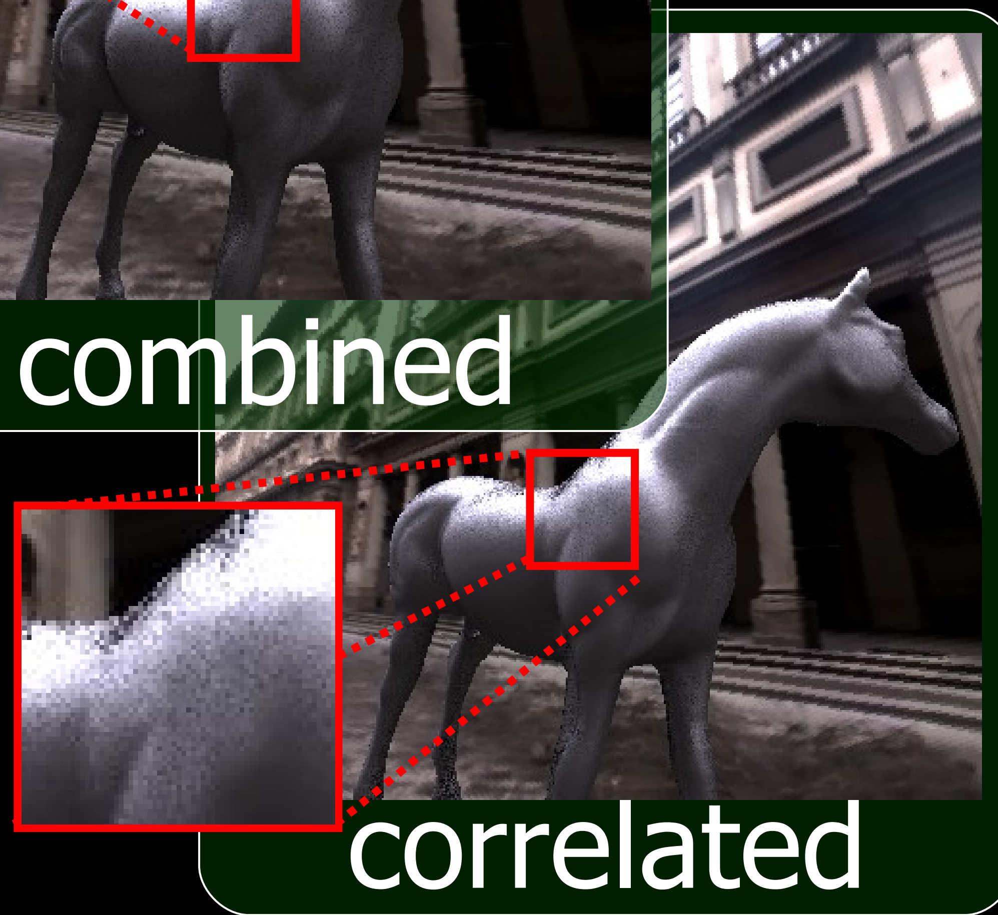


orrelated

Environme… …ults







Thank you